Appendix A

Skogestad's method for PID tuning

The design principle of Skogestad's method is as follows. The control system *tracking transfer function* $T(s)$, which is the transfer function from the reference or setpoint to the (filtered) process measurement, is *specified* as a first order transfer function with time delay:

$$T(s) = \frac{y_{mf}(s)}{y_{m_{SP}}(s)} = \frac{1}{T_C s + 1} e^{-\tau s} \tag{A.1}$$

where T_C is the time constant of the control system which *the user must specify*, and τ is the process time delay which is *given* by the process model (the method can however be used for processes without time delay, too). Figure A.1 shows the response in y_{mf} after a step in the setpoint $y_{m_{SP}}$ for (A.1). (This response will be achieved only approximately because Skogestad's method is based on some simplifying assumptions.)

Skogestad's tuning formulas for several processes are shown in Table A.1. Note that the process transfer function actually includes the sensor. Hence, $H(s)$ is the transfer function from control signal to measurement signal.

Unless you have reasons for a different specification, Skogestad suggests

$$T_C = \tau \tag{A.2}$$

to be used for T_C in Table A.1. If the time delay τ is zero, you must yourself define a reasonable value of T_C.

Note that Skogestad's formulas assumes a *serial* PID function. If your controller actually implementes a *parallel* PID controller (as in the PID

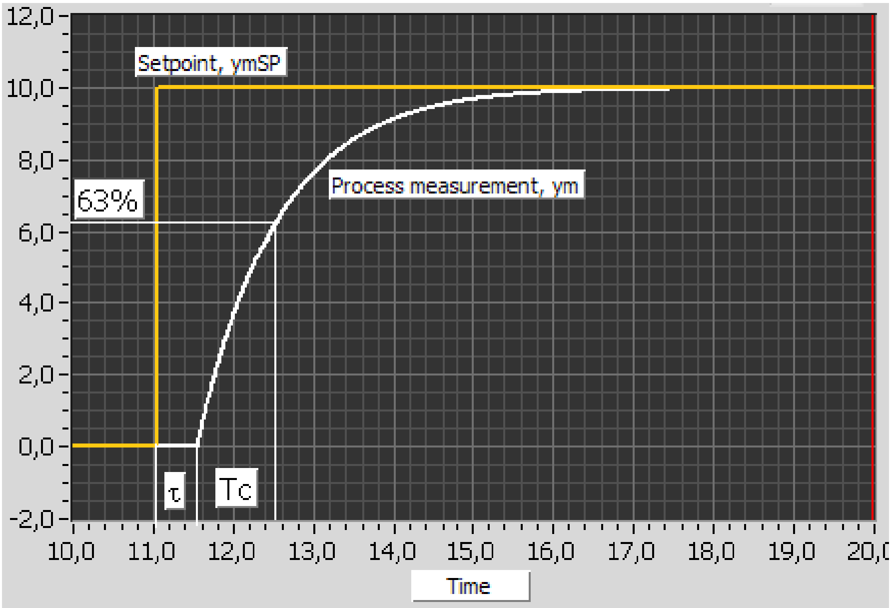

Figure A.1: Step response of the specified tracking transfer function (A.1) in Skogestad's PID tuning method

controllers in LabVIEW PID Control Toolkit and in the Matlab/Simulink PID controllers), you should transform from serial PID settings to parallell PID settings. If you do not implement these transformations, the control system may behave unnecessarily different from the specified response.[1] The serial-to-parallel transformations are as follows:

$$K_{p_p} = K_{p_s}\left(1 + \frac{T_{d_s}}{T_{i_s}}\right) \tag{A.3}$$

$$T_{i_p} = T_{i_s}\left(1 + \frac{T_{d_s}}{T_{i_s}}\right) \tag{A.4}$$

$$T_{d_p} = T_{d_s}\frac{1}{1 + \frac{T_{d_s}}{T_{i_s}}} \tag{A.5}$$

where K_{p_p}, T_{i_p}, and T_{d_p} are parameters of the following parallel PID controller (given on transfer function form):

$$u(s) = \left[K_{p_p} + \frac{K_{p_p}}{T_{i_p}s} + K_{p_p}T_{d_p}s\right]e(s) \tag{A.6}$$

[1]The transformations are important because the integral time and the derivative time are equal, cf. (15.38) and (15.39). If the integral time is substantially larger than the derivative time, e.g. four times larger, the transformations are not necessary.

$H(s)$	K_p	T_i	T_d
$\frac{K}{s}e^{-\tau s}$	$\frac{1}{K(T_C+\tau)}$	$k_1\left(T_C+\tau\right)$	0
$\frac{K}{Ts+1}e^{-\tau s}$	$\frac{T}{K(T_C+\tau)}$	$\min\left[T,\, k_1\left(T_C+\tau\right)\right]$	0
$\frac{K}{(Ts+1)s}e^{-\tau s}$	$\frac{1}{K(T_C+\tau)}$	$k_1\left(T_C+\tau\right)$	T
$\frac{K}{(T_1s+1)(T_2s+1)}e^{-\tau s}$	$\frac{T_1}{K(T_C+\tau)}$	$\min\left[T_1,\, k_1\left(T_C+\tau\right)\right]$	T_2
$\frac{K}{s^2}e^{-\tau s}$	$\frac{1}{4K(T_C+\tau)^2}$	$4\left(T_C+\tau\right)$	$4\left(T_C+\tau\right)$

Table A.1: Skogestad's formulas for PI(D) tuning. Standard value of k_1 is 4, but a smaller value, e.g. $k_1 = 1.44$ can give faster disturbance compensation. For the second order the process T_1 is the largest and and T_2 is the smallest time constant. (min means the minimum value.)

and K_{p_s}, T_{i_s}, and T_{d_s} are parameters of the following serial PID controller:

$$u(s) = K_{p_s}\frac{\left(T_{i_s}s+1\right)\left(T_{d_s}s+1\right)}{T_{i_s}s}e(s) \tag{A.7}$$