Høgskolen i Buskerud. Finn Haugen (finn.haugen@hibu.no).

# Løsning til eksamen i SEKY3322 Kybernetikk 3

Tid: 2. april 2009. Varighet 5 timer. Vekt i sluttkarakteren: 70%.

Hjelpemidler: Ingen trykte eller håndskrevne hjelpemidler. Kalkulator ikke tillatt.

Bakerst i oppgavesettet er det en formelsamling.

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1. (10% vekt) Skriver først modellen på standard regresjonsform:

$$h(k) - h(k-1) = \begin{bmatrix} -\sqrt{h(k-1)} & u(k-1) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{1}$$

Den totale modellen blir

$$\underbrace{\begin{bmatrix} h(1) - h(0) \\ h(2) - h(1) \\ h(3) - h(2) \\ h(4) - h(3) \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} -\sqrt{h(0)} & u(0) \\ -\sqrt{h(1)} & u(1) \\ -\sqrt{h(2)} & u(2) \\ -\sqrt{h(3)} & u(3) \end{bmatrix}}_{\Phi} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\theta} \tag{2}$$

2. (20) Innfører $x_1 = n$ og $x_2 = L$ og $y = x_1 = n$. Differensiallikningen kan da uttrykkes som

$$T\dot{x}_1(t) + x_1(t) = K[u(t) - x_2(t)] \tag{3}$$

eller

$$\dot{x}_1(t) = -\frac{1}{T}x_1(t) + \frac{K}{T}[u(t) - x_2(t)] \tag{4}$$

Antar dessuten at $x_2 = L$ er langsomt varierende eller nesten konstant:

$$\dot{x}_2(t) = 0 \tag{5}$$

Målelikningen blir

$$y(t) = x_1(t) \tag{6}$$

Diskretisering med Eulers forovermetode gir

$$\frac{x_1(k+1) - x_1(k)}{h} = -\frac{1}{T}x_1(k) + \frac{K}{T}[u(k) - x_2(k)] \tag{7}$$

$$\frac{x_2(k+1) - x_2(k)}{h} = 0 \tag{8}$$

Den tidsdiskrete tilstandsrommodellen blir, med prosessforstyrrelser og målestøy inkludert,

$$x_1(k+1) = x_1(k) - \frac{h}{T}x_1(k) + \frac{hK}{T}[u(k) - x_2(k)] + v_1(k) \quad (9)$$

$$x_2(k+1) = x_2(k) + v_2(k) \quad (10)$$

$$y(k) = x_1(x) + w(k) \quad (11)$$

*Kalman-filteret:*

Innovasjonsprosessen ($x_1(k)$ er måling):

$$\underline{\underline{y(k) = x_1(k)}} \quad (12)$$

Korrektorestimatene:

$$\underline{\underline{\begin{aligned} x_{1_c}(k+1) &= x_{1_p}(k) + K_{11}e(k) \\ x_{2_c}(k+1) &= x_{2_p}(k) + K_{21}e(k) \end{aligned}}} \quad (13)$$

Prediktorestimatene:

$$\underline{\underline{\begin{aligned} x_{1_p}(k+1) &= x_{1_c}(k) - \tfrac{h}{T}x_{1_c}(k) + \tfrac{hK}{T}[u(k) - x_{2_c}(k)] \\ x_{2_p}(k+1) &= x_{2_c}(k) \end{aligned}}} \quad (14)$$

3. (5) Hurtigere respons i estimatet for tilstandsvariabel nr. $n$ kan oppnås ved å øke støykovarianselementet $Q_{nn}$ for denne tilstandsvariabelen (dette medfører økt verdi av den tilsvarende Kalmanfilterforsterkningen).

   Ulempe: Estimatet vil inneholde med støy som stammer fra målestøyen (den økte verdien av Kalmanfilterforsterkningen vil øke påvirkningen fra målestøyen på estimatet).

4. (5) For eksempel:

$$\underline{\underline{\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} -0,5 & 0 & 1 \\ -3 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}}_{B} u(k)}} \quad (15)$$

$$\underline{\underline{\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{C} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}_{D} u(k)}} \quad (16)$$

5. (5) $y$ skal beregnes fra $u$ slik:

$$\underline{\underline{y = Gu + C}} \tag{17}$$

der $G$ og $C$ kan beregnes fra de oppgitte formlene

$$m_y = Gm_u + C \tag{18}$$

og

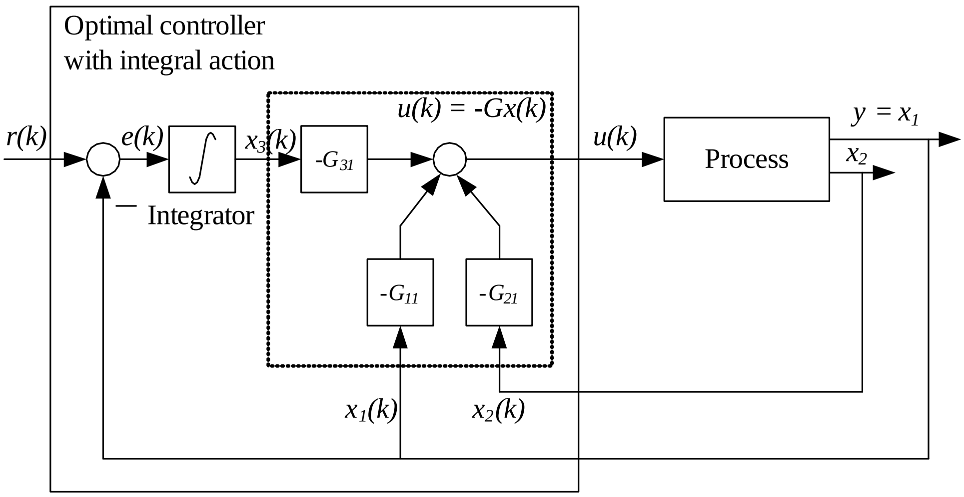
$$\sigma_y^2 = G^2\sigma_u^2 \tag{19}$$

der $m_u = 0$, $m_y = 3$, $\sigma_u^2 = 1$, $\sigma_y^2 = 4$. Vi får

$$\underline{\underline{G = 2}} \tag{20}$$

$$\underline{\underline{C = 3}} \tag{21}$$

6. (15) Se figur 1.




Figur 1:

7. (5) For å oppnå jevnere pådragsbruk kan elementer i $R$-matrisen økes. Dette vil øke kostnaden på pådragsendringer, dvs. at pådragsendringene blir mindre, dvs. jevnere pådragsbruk.

8. (10) Vi løser mht. pådraget og setter referansen inn for prosessvariabelen. Resultatet blir

$$u_f = \underbrace{\frac{-\dot{r} + ar}{b}}_{F_r} + \underbrace{\frac{c}{b}}_{F_f} v \tag{22}$$

Foroverkoplingsfunksjonen er høyresiden i ovenstående likning. $u_f$ er foroverkoplingpådraget. Det kan antas at referansen $r$ er kjent. Forstyrrelsen $v$ må måles.

9. (15) Siden begrepet *intern PI-regulator* – og ikke intern PID-regulator – står i oppgaven, kan det antas at det er standard-versjonen av feedback linearization som skal utledes.

   Utledningen nedenfor er stort sett kopiert fra en lærebok/kompendium i emnet. Det forventes ikke at studentene gir en like detaljert besvarelse, men hovedpunktene skal være med i en fullgod besvarelse.

   It assumed that the process model is a state space model on the following form:

$$\dot{x} = f(x, v) + B(x, v) \cdot u \tag{23}$$

   or, simpler,

$$\dot{x} = f + Bu \tag{24}$$

   $x$ is the state vector, $v$ is the disturbance vector, and $u$ is the control vector. $f$ is a vector of scalar functions, and $B$ is a matrix of scalar functions. Assume that the output vector is

$$y = x \tag{25}$$

   By taking the derivative of (25) and using (24) we obtain the following differential equation describing the process output vector:

$$\dot{y} = f + Bu \tag{26}$$

   Assume that $r_y$ is the reference (or setpoint) of $y$. With the above assumptions, we derive the control function as follows: We start by defining the *transformed control vector* as

$$z \overset{\text{def}}{=} f + Bu \tag{27}$$

   Then (26) can be written as

$$\dot{y} = z \tag{28}$$

   which are $n$ decoupled or independent *integrators* ($n$ is the number of state variables), because $y(t) = \int_0^t z \, d\tau$. The transfer function from $z$ to $y$ is

$$\frac{y(s)}{z(s)} = \frac{1}{s} \tag{29}$$

   We will now derive the control function for this integrator process, and thereafter derive the final control function. How can you control an integrator? With feedback and feedforward! A good choice for the *feedback controller* is a PI controller (proportional plus integral) because the controller should contain integral action to ensure zero steady-state control error in the presence of unmodelled disturbances (and there are such in a real system). The proportional action is necessary to get a stable control system (if a pure integral controller acts on an integration process the closed loop system becomes marginally stable, i.e. it is pure oscillatory). The multiloop feedback PI controller is

$$z_{\text{fb}} = K_p e + K_i \int_0^t e d\tau \tag{30}$$

where $e$ is the control error:

$$e \stackrel{\text{def}}{=} r_y - y \tag{31}$$

In addition to the PI feedback action the controller should contain *feedforward* from the reference $r_y$ to get fast reference tracking when needed (assuming the reference is varying). The feedforward control function can be derived by substituting the process output $y$ in the process model (28) by $r_y$ and then solving for $y$, giving

$$z_{\text{ff}} = \dot{r}_{y_f} \tag{32}$$

where index $f$ indicates lowpass filter which may be of first order. A pure time differentiation should not be implemented because of noise amplification by the differentiation. Therefore the reference should be lowpass filtered before its time derivative is calculated.

The control function for the process (28) based on the sum of the feedback control function and the feedforward control function is as follows:

$$\begin{aligned} z &= z_{\text{fb}} + z_{\text{ff}} && (33) \\ &= \underbrace{K_p e + K_i \int_0^t e d\tau}_{z_{\text{fb}}} + \underbrace{\dot{r}_{y_f}}_{z_{\text{ff}}} && (34) \end{aligned}$$

Deriving the final control function, that is, the formula for the control vector $u$: From (27) we get

$$u = B^{-1} (z - f) \tag{35}$$

Here we use (34) to get the final control function:

$$\underline{u = B^{-1} \left( K_p e + K_i \int_0^t e d\tau + \dot{r}_{y_f} - f \right)} \tag{36}$$

How to tune the PI controller included in the control function (36)? In (36) $K_p$ and $K_i$ are diagonal matrices:

$$K_p = \begin{bmatrix} K_{p_1} & 0 & \cdots & 0 \\ 0 & K_{p_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_{p_n} \end{bmatrix} = \mathrm{diag}(K_{p_1}, K_{p_2}, \cdots, K_{p_n}) \qquad (37)$$

$$K_i = \begin{bmatrix} K_{i_1} & 0 & \cdots & 0 \\ 0 & K_{i_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_{i_n} \end{bmatrix} = \mathrm{diag}(K_{i_1}, K_{i_2}, \cdots, K_{i_n}) \qquad (38)$$

where the scalar values are

$$K_{i_j} = \frac{K_{p_j}}{T_{i_j}} \qquad (39)$$

where $K_{p_j}$ is the proportional gain and $T_{i_j}$ is the integral time of control loop no. $j$.

$K_{p_j}$ and $T_{i_j}$ can be calculated in several ways. *Skogestad's method* is one option. Skogestad's method is reviewed below. The design principle of Skogestad's method is as follows. The control system *tracking transfer function* $T(s)$, which is the transfer function from the reference or setpoint to the (filtered) process measurement, is *specified* as a first order transfer function with time delay:

$$T(s) = \frac{y_{mf}(s)}{y_{m_{SP}}(s)} = \frac{1}{T_C s + 1} e^{-\tau s} \qquad (40)$$

where $T_C$ is the time constant of the control system which *the user must specify*, and $\tau$ is the process time delay which is *given* by the process model.

Skogestad's tuning formulas for an integrator with gain equal to one and time-delay equal to zero are as follows:

$$\underline{K_{p_j} = \frac{1}{T_{C_j}}} \qquad (41)$$

and

$$\underline{T_{i_j} = k_j T_{C_j}} \qquad (42)$$

where $T_{C_j}$ is the specified time constant of feedback loop no. $j$, and $k_j$ is a coefficient that can be set to 1.44.

10. (5) You can *simulate the control system*. In the simulator you can include model errors by using *different* models in the control function and in the process in the simulator.

11. (5) Du kan la regulatoren bruke tilstandsestimatene i Kalmanfilteret til pådragsberegningen. Når sensoren faller ut, skal Kalmanfilteret oppdatere prediktorestimatene i Kalmanfilteret, mens korrektorestimatene (målebaserte estimater) ikke skal oppdateres siden målingene jo har falt ut. Regulatoren skal fortsette å benytte estimatene, som nå i praksis vil være prediktorestimatene, som igjen vil være simulerte tilstander (siden det ikke er noe målebasert korreksjon av estimatene). Dermed kan reguleringen fortsette så å si som normalt, i hvert fall en stund, til tross for sensorutfall.