Høgskolen i Buskerud
Finn Haugen (finn.haugen@hibu.no)
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1 Multivariable control

1.1 Introduction

Multivariable processes has more than one input variables or ore than one output variables. Here are a few examples of multivariable processes:

- A heated liquid tank where both the level and the temperature shall be controlled.
- A distillation column where the top and bottom concentration shall be controlled.
- A robot manipulator where the positions of the manipulators (arms) shall be controlled.
- A chemical reactor where the concentration and the temperature shall be controlled.
- A head box (in a paper factory) where the bottom pressure and the paper mass level in the head box shall be controlled.

To each variable (process output variable) which is to be controlled a setpoint is given. To control these variables a number of control variables are available for manipulation by the controller function.

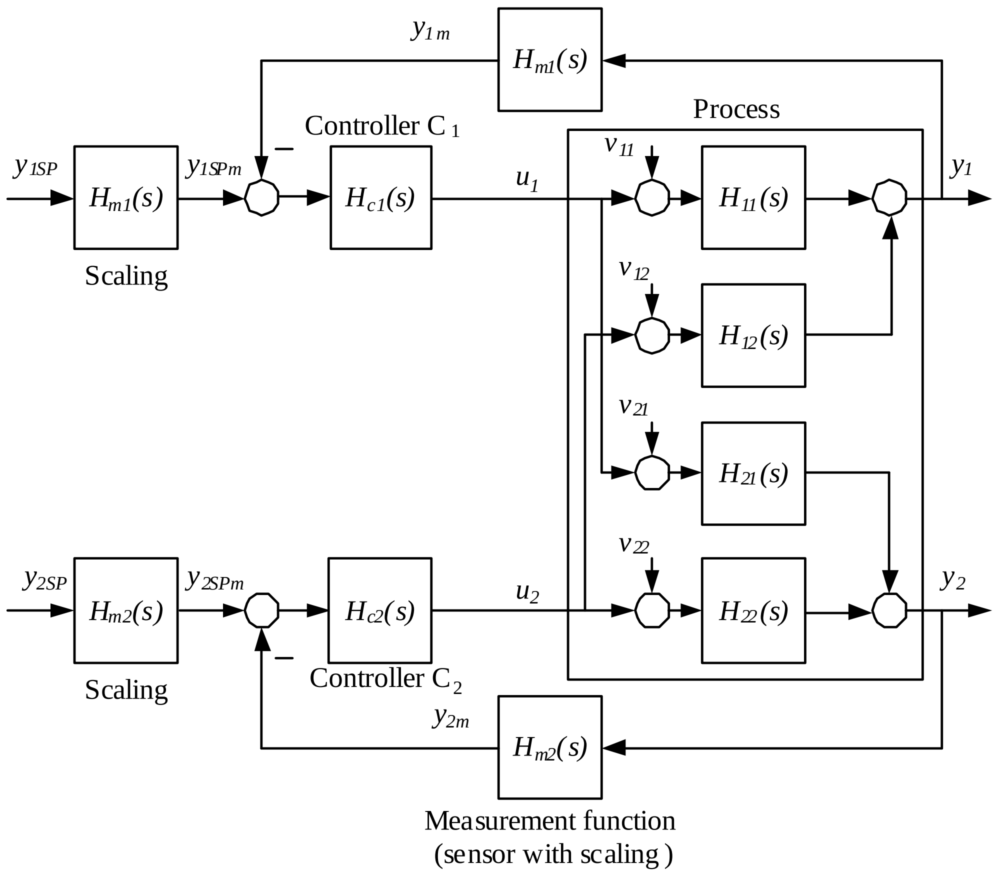
Multivariable processes can be difficult to control if there are *cross couplings* in the process, that is, if one control variable gives a response in several process output variables. There are mainly two problems of controlling a multivariable process if these cross couplings are not counteracted by the multivariable controller:

- A change in one setpoint will cause a response in each of the process output variables, not only in the output variable corresponding to the setpoint.
- Assuming that ordinary single loop PID control is used, a controller will "observe" a complicated dynamic system which consists of the multivariable process *with* all control loops! This can make it difficult to tune each of the PID controllers, and the stability robustness of the control system may be small.

The following section describes the most common way to control multivariable processes – namely single loop control with PID controllers.

1.2 Single loop control with PID controllers

The simplest yet most common way to control a multivariable process is using *single loop control* with PID controllers. There is one control loop for each process output variable which is to be controlled. The control system structure is shown in Figure 1, where subsystems are represented by transfer functions although these subsystems are generally non-linear dynamic systems. Since this process has two control variables and two process output variables, we say that the process is a 2x2 multivariable process.



Figur 1: Single loop control of a 2x2 multivariable process

1.2.1 Pairing of process output variables and control variables

In single loop control of a multivariable process we must determine the pairing of process output variable (its measurement) and control variable (via the PID controller). A natural rule for choosing this pairing is as follows: *The strong process couplings (from control variable to process output variable) should be contained in the control loops.* Following this rule is an effective use of the control variable, and supports stability robustness against variations of the dynamic properties in other parts of the control system. Figure 1 shows the correct control system structure if there are strong couplings in $H_{11}(s)$ and in $H_{22}(s)$.

In most cases it is easy to determine the strong pairings. One example is a heated liquid tank where both level and temperature is to be controlled. The two control variables are power supply via a heating element and liquid supply. This process is multivariable with cross couplings since both power supply (control variable 1) and liquid supply (control variable 2) influences both process output variables (level and temperature). (The level is influenced by the power supply through liquid expansion due to temperature increase.) In this case the process output variable/control variable pairing is obvious: Level $\leftrightarrow$ Power and Temperature $\leftrightarrow$ Liquid flow, right?[1]

There are model based methods for analysis of process couplings, as RGA-analysis (Relative Gain Array) and singular value analysis, and there are multivariable control functions that includes a decoupler that counteracts the couplings in the process. A reference for such methods is e.g. [1].

1.2.2 Controller tuning

In the tuning procedures below you can try the Ziegler-Nichols' closed loop method or the P-I-D tuning method.

According to [1] a widely used procedure for tuning the PID controllers in single loop multivariable control is as follows:

Procedure 1:

1. Tune the controller in each of the loops in turn with all the other controllers in manual mode.
2. Close all the loops (set all controllers in automatic mode).

[1] Wrong :-)

3. If there are stability problems, reduce the gain and/or increase the integral time of the controllers in the least important loops.

An alternative procedure [1] for cases where the control of one specific process variable is more important than the control of other variables is as follows:

Procedure 2:

1. Tune the controller of the most important loop. The other controllers are set in manual mode.
2. Tune the other controllers in sequence, with the tuned controllers set in automatic mode.
3. If there are stability problems, reduce the gain and/or increase the integral time of the controllers in the least important loops.

Example 1 ***Single loop multivariable control***

See Figure 1. The process transfer functions are on the form

$$H_{ij}(s) = \frac{y_i(s)}{u_j(s)} = \frac{K_{ij}}{T_{ij}s+1}e^{-\tau_{ij}s} \tag{1}$$

with these parameters:

$$K_{11} = 1;\ T_{11} = 1;\ \tau_{11} = 0.5 \tag{2}$$

$$K_{12} = 0.5;\ T_{12} = 1;\ \tau_{12} = 0.5 \tag{3}$$

$$K_{21} = 0.5;\ T_{21} = 1;\ \tau_{21} = 0.5 \tag{4}$$

$$K_{22} = 1;\ T_{22} = 1;\ \tau_{22} = 0.5 \tag{5}$$

Thus, there are cross couplings "both ways" in the process since both K_{12} and K_{21} are different from zero.

The measurement transfer functions are $H_{m_1}(s) = 1 = H_{m_2}(s)$. The controllers are PID controllers tuned according to Procedure 1 described above (with the Ziegler-Nichols' closed loop method). The tuning gives

$$K_{p_1} = 2.0;\ T_{i_1} = 0.9;\ T_{d_1} = 0.23 \tag{6}$$

$$K_{p_2} = 2.0;\ T_{i_2} = 0.9;\ T_{d_2} = 0.23 \tag{7}$$

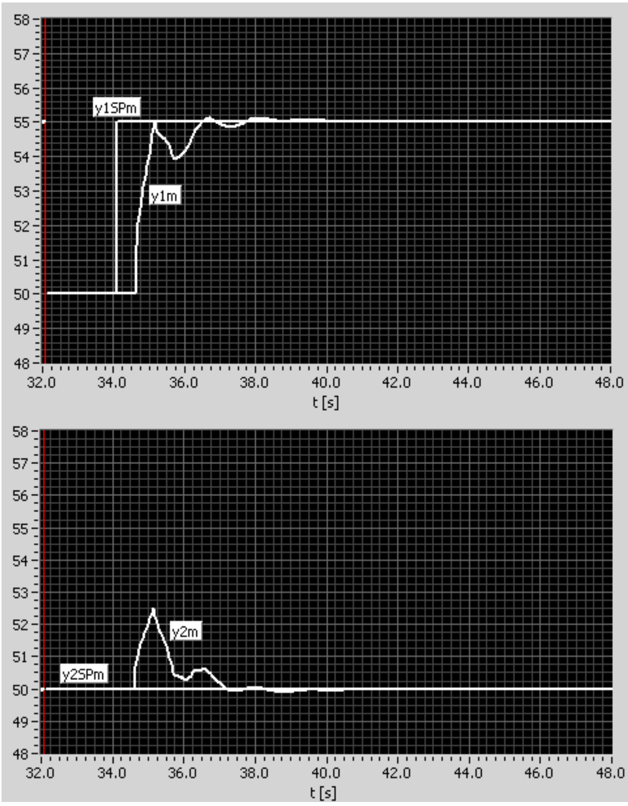
However, simulations shows that the multivariable control system actually is unstable using the above PID settings. So, re-tuning was necessary.

Decreasing the proportional gains from 2.0 to 1.4 was sufficient in this case. The final settings are

$$K_{p_1} = 1.4;\ T_{i_1} = 0.9;\ T_{d_1} = 0.23 \tag{8}$$

$$K_{p_2} = 1.4;\ T_{i_2} = 0.9;\ T_{d_2} = 0.23 \tag{9}$$

Figure 2 shows simulated responses in y_{1_m} and y_{2_m} due to a step in y_{1SP_m}. As expected, the setpoint step gives a cross response in y_{2_m}. The stability



Figur 2: Example 1: Single loop multivariable control. Simulated responses in y_{1m} and y_{2_m} due to a step in the setpoint $y_{1_{SPm}}$.

of the control system seems to be acceptable.

[End of Example 1]

Referanser

[1] Seborg, Edgar, and Mellichamp, **Process Dynamic and Control**, Wiley, 2. edition, 2004