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Emne E0404 Prosjekt: Reguleringssystem. Øving 2

Exercise 0.1 ***Discretizing the derivative part of a PID controller***

The transfer function from control error e to derivative term u_d of a PID control is

$$u_d(s) = \frac{K_p T_d s}{T_f s + 1} e(s) \tag{1}$$

where K_p (controller gain), T_d (derivative time) and T_f (filtering time) are parameters (constants). Discretize the derivative part, i.e., find a formula for calculating $u_d(t_k)$. Use the Euler Backward method in the discretization.

Exercise 0.2 ***Calculating transfer function from difference equation***

Given the following difference equation which is a so called FIR filter (Finite Impulse Reponse) or a MA filter (Moving Avarage) filter:

$$y(k) = \frac{u(k) + u(k-1) + u(k-2) + u(k-3) + u(k-4)}{5} \tag{2}$$

Find the tranfer function, $H(z)$, from filter input u to filter output y. Write it in non-negative exponents of z.

Solution 0.1

From (1) we get

$$(T_f s + 1)\, u_d(s) = K_p T_d s e(s) \tag{3}$$

or

$$T_f s u_d(s) + u_d(s) = K_p T_d s e(s) \tag{4}$$

Taking the inverse Laplace transform gives

$$T_f \dot{u}_d(t) + u_d(t) = K_p T_d \dot{e}(t) \tag{5}$$

Approximating the time derivatives with their Backward approksimations, gives

$$T_f \frac{u_d(t_k) - u_d(t_{k-1})}{h} + u_d(t) = K_p T_d \frac{e(t_k) - e(t_{k-1})}{h} \tag{6}$$

Solving for $u_d(t_k)$ gives

$$\underline{\underline{u_d(t_k) = \frac{T_f}{T_f + h} u_d(t_{k-1}) + \frac{K_p T_d}{T_f + h} \left[e(t_k) - e(t_{k-1}) \right]}} \tag{7}$$

Solution 0.2

Taking the z-transform of both sides of (2) gives

$$\begin{aligned} y(z) &= \frac{u(z) + z^{-1}u(z) + z^{-2}u(z) + z^{-3}u(z) + z^{-4}u(z)}{5} && (8) \\ &= \frac{1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}}{5} u(z) && (9) \\ &= \underbrace{\frac{z^4 + z^3 + z^2 + z + 1}{5z^4}}_{H(z)} u(z) && (10) \end{aligned}$$