Høgskolen i Telemark/Finn Haugen (finn.haugen@hit.no)/24.1.07

Emne E0404 Prosjekt: Reguleringssystem. Øving 3

Exercise 0.1 ***Block diagram manipulation***

Figure 1 shows a block diagram of a feedback control system.

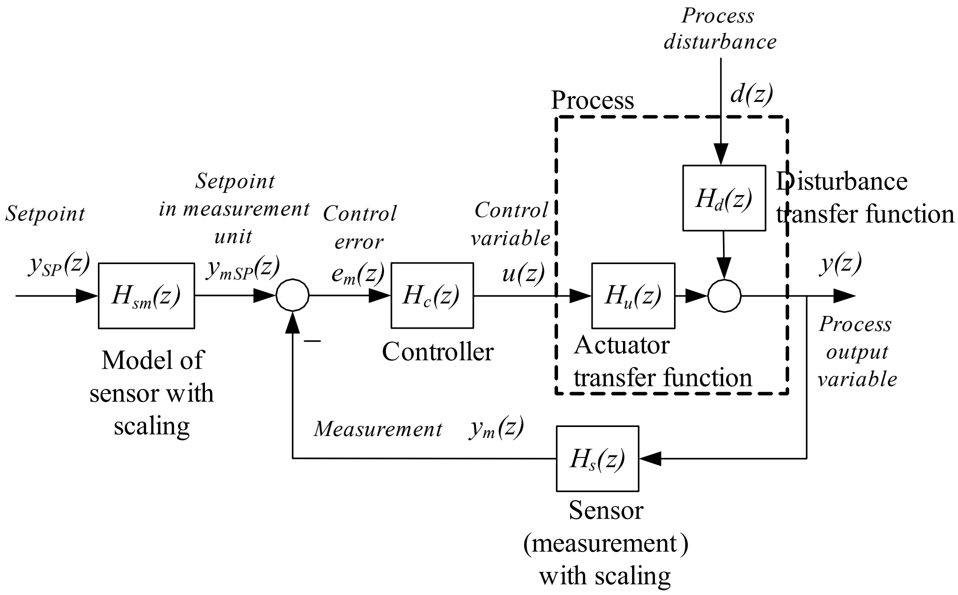

Figure 1: Block diagram of a feedback control system

1. Find the transfer function $T(z)$ from setpoint y_{SP} to process output variable y by manipulating the block diagram. ($T(z)$ is denoted the tracking transfer function of the control system.)

2. Actually you do not have to manipulate the block diagram to calculate the transfer function from it. You can just write down the proper equations from the block diagram, and then find the transfer function from these equations. Find $T(z)$ by writing the equation for $y(z)$ from the block diagram and then calculating $T(z)$ from the equation. (You may drop the argument z while doing the calculations to save some writing. You may also set the input variable $d = 0$ since the task is not to find the transfer function from d to y.)

   Tip: $y = H_u u = H_u H_c e = \cdots$.

Exercise 0.2 ***Poles and zeros***

Given the following transfer function:

$$H(z) = \frac{bz^{-2} + z^{-1}}{1 - az^{-1}} \tag{1}$$

Calculate the poles and the zeros of the transfer function.

Exercise 0.3 ***Delay***

Given the following transfer function:

$$H(z) = \frac{b_1 z + b_0}{a_4 z^4 + a_3 z^3} = \frac{y(z)}{u(z)} \tag{2}$$

Find the delay between input u and y (expressed in number of time steps).

Exercise 0.4 ***Frequency response***

Find the frequency response of the following transfer function:

$$H(z) = \frac{hz}{z-1} \tag{3}$$

(You do not have to calculate the amplitude gain function and the phase lag function.)

Solution 0.1

1. Figure 2 shows the individual steps of the block diagram manipulation. The transfer function becomes

$$T(z) = \frac{y(z)}{y_{SP}(z)} = \frac{H_{sm}(z)H_c(z)H_u(z)}{1 + H_s(z)H_c(z)H_u(z)} \tag{4}$$

2. From the block diagram in Figure 1:

$$\begin{aligned} y &= H_u u \\ &= H_u H_c e \\ &= H_u H_c (y_{mSP} - y_m) \\ &= H_u H_c (H_{sm} y_{SP} - H_s y) \end{aligned}$$

Solving this expression for y yields

$$y(z) = \underbrace{\frac{H_{sm}(z)H_c(z)H_u(z)}{1 + H_s(z)H_c(z)H_u(z)}}_{=T(z)} y_{SP}(z) \tag{5}$$

which gives the same transfer function $T(s)$, which is the same as found by block diagram manipulation in Problem 1 above.

Solution 0.2

It is convenient to start by rewriting the transfer function as follows:

$$H(z) = \frac{z^{-2}b + z^{-1}}{1 - az^{-1}} \cdot \frac{z^2}{z^2} = \frac{z + b}{z^2 - az} = \frac{z + b}{(z - a)\, z} \tag{6}$$

Thus, $\underline{\underline{\text{the zero is } -b}}$, and $\underline{\underline{\text{the poles are } a \text{ and } 0}}$.

Solution 0.3

The delay is the same as the number of z^{-1}-factors. By writing the transfer function as

$$H(z) = \frac{b_1 z + b_0}{a_4 z^4 + a_3 z^3} = \frac{b_1 z + b_0}{a_4 z + a_3} \cdot \frac{1}{z^3} = \frac{b_1 z + b_0}{a_4 z + a_3} \cdot \left(z^{-1}\right)^3 \tag{7}$$

we see that $\underline{\underline{\text{the delay is } 3}}$.

Solution 0.4

The frequency response is found by substituting z with $e^{j\omega h}$ in the transfer function:

$$\underline{\underline{H(e^{j\omega h}) = \frac{h e^{j\omega h}}{e^{j\omega h} - 1}}} \tag{8}$$

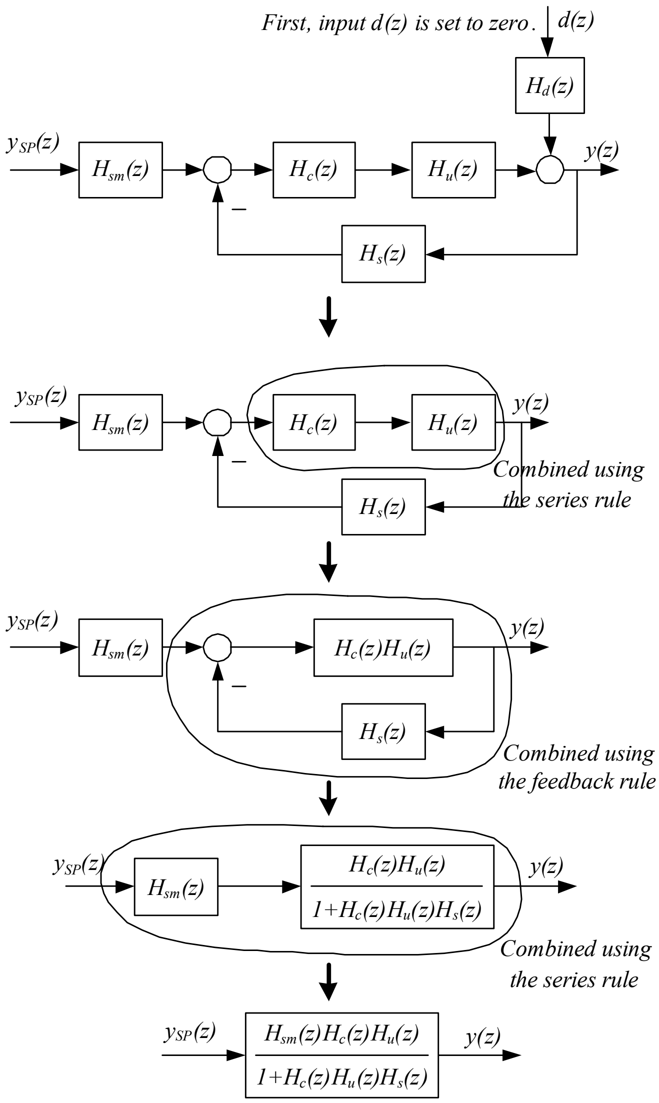
First, input d(z) is set to zero.
d(z)
Hd(z)
ySP(z)
Hsm(z)
Hc(z)
Hu(z)
y(z)
Hs(z)
Combined using
the series rule
Hc(z)Hu(z)
Combined using
the feedback rule
Hc(z)Hu(z) / 1+Hc(z)Hu(z)Hs(z)
Combined using
the series rule
Hsm(z)Hc(z)Hu(z) / 1+Hc(z)Hu(z)Hs(z)

Figure 2: