Inserting the above two transfer functions into (9.6) yields the following feedforward transfer function:

$$H_f(s) = \frac{u_{fd}(s)}{v(s)} = \frac{-H_v(s)}{H_u(s)} = -\frac{K_v}{K_u} \cdot \frac{T_u s + 1}{T_v s + 1} \tag{9.11}$$

which is a *lead-lag-function*, which is available as a functional block in most commercial controllers. If $T_u > T_v$ the transfer function has lead-effect because the phase function of the frequency response has positive value which means that the response of the lead-lag function is phase leading (is ahead in phase). If $T_u < T_v$ the transfer function has lag-effect because the phase function of the frequency response has negative value which means that the response of the function is phase lagging (is behind in phase).

[End of Example 9.2]

Example 9.3 ***Feedforward control from the setpoint***

For DC-motors the transfer function from the control variable u to the angular velocity y is approximately

$$\frac{y(s)}{u(s)} = H_u(s) = \frac{K}{(Ts+1)\,s} \tag{9.12}$$

(that is, a first order system with integrator). The feedforward function $H_{fSP}(s)$ in (9.6) becomes

$$H_{fSP}(s) = \frac{u_{fSP}(s)}{(s)} = \frac{1}{H_u(s)} = \frac{(Ts+1)\,s}{K} = \frac{T}{K}s^2 + \frac{1}{K}s \tag{9.13}$$

which in the time-domain corresponds to

$$u_{fSP}(t) = \frac{T}{K}\ddot{y}_{SP}(t) + \frac{1}{K}\dot{y}_{SP}(t) \tag{9.14}$$

To avoid numerical problems of calculating the derivatives in (9.14) the setpoint y_{SP} may be chosen to be sufficiently smooth. For example, setpoint changes could be in the form of parabolic functions of time since this signal has a continuous second order time derivative. Another solution is to use a lowpass filter in the setpoint path, as shown in Figure 7.18.

[End of Example 9.3]

9.2 Cascade control

From earlier chapters we know that a control loop compensates for disturbances so that the control error is small despite the disturbances. If the controller has integral action the steady-state control error is zero. What more can we wish? In some applications it may be desirable if the transient time progression of the error is faster, so that e.g. the IAE index, cf. Section 2.8, is smaller. This can be achieved by *cascade control*, see Figure 9.7.

In a cascade control system there is one or more control loops inside the *primary loop*, and the controllers are in cascade. There is usually one, but

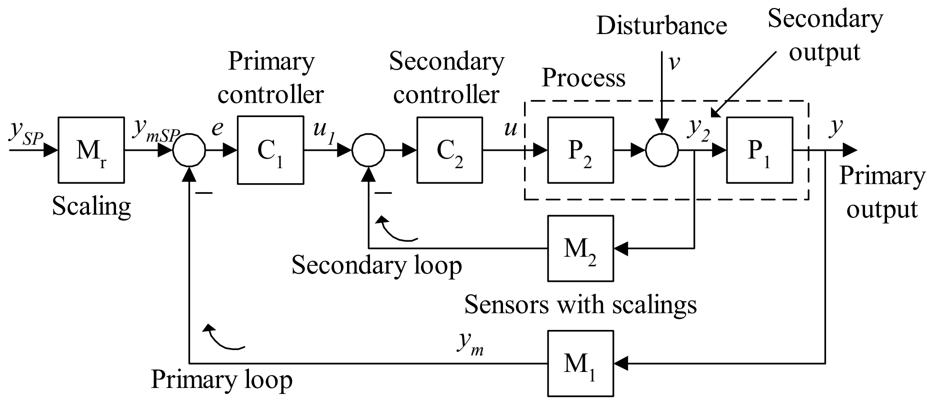

Figure 9.7: Cascade control system

there may be two and even three internal loops inside the primary loop. The (first) loop inside the primary loop is called the *secondary loop*, and the controller in this loop is called the *secondary controller* (or slave controller). The outer loop is called the *primary loop*, and the controller in this loop is called the primary controller (or master-controller). The control signal calculated by the primary controller is the setpoint of the secondary controller.

In most applications the purpose of the secondary loop is to compensate quickly for the disturbance so that its response in the primary output variable of the process is small. For this to happen the secondary loop must register the disturbance. This is done with the sensor M_2 in Figure 9.7.

In addition to getting better disturbance compensation cascade control may give a more *linear relation* between the variables u_1 and y_2, see Figure 9.7 than with usual single loop control. In many applications process part 2 (P_2 in Figure 9.7) is the actuator. In this case the secondary loop can be regarded as a new actuator having better linearity (or proportionality). One example is a control valve where the secondary loop is a flow control loop. With this secondary loop there is a more linear relation between the control signal and the flow than without such a loop. The better linearity may make the tuning of the primary controller (performing e.g. level or temperature control) easier and with more robust stability properties.

The improved control with cascade control can be explained by the increased information about the process – there is at least one more measurement. It is a general principle that the more information you have about the process to be controlled, the better it can be controlled. Note however, that there is still only one control variable to the process, but it is based on two or more measurements.

Since cascade control requires at least two sensors a cascade control system is somewhat more expensive than a single loop control system. Except for cheap control equipment, commercial control equipment are typically prepared for cascade control, so no extra control hardware or software is required.

In which frequency range is the secondary loop effective for compensation for disturbances? This is given by the bandwidth of the secondary loop. A proper bandwidth definition here is the -11 dB-the bandwidth ω_s of the sensitivity function, $S_2(s)$, of the secondary loop, cf. Chapter 6.3.4. $S_2(s)$ is

$$S_2(s) = \frac{1}{1 + L_2(s)} = \frac{1}{1 + H_{c_2}(s)H_{u_2}(s)H_{m_2}(s)} \tag{9.15}$$

where $L_2(s)$ is the loop transfer function of the secondary loop. $H_{c_2}(s)$ is the transfer function of the secondary controller. $H_{u_2}(s)$ is the transfer function from the control variable to the secondary process output variable, y_2. $H_{m_2}(s)$ is the measurement transfer function of the secondary sensor.

As explained above cascade control can give substantial compensation improvement. Cascade control can also give improved tracking of a varying setpoint, but only if the secondary loop has faster dynamics than the process part P_2 itself, cf. Figure 9.7, so that the primary controller "sees" a faster process. If there is a time delay in P_2, the secondary loop will not be faster than P_2 (this is demonstrated in Example 9.4). In most applications improved compensation – not improved tracking – is the main purpose of cascade control.

The secondary controller is typically a P controller or a PI controller. The derivative action is usually not needed to speed up the secondary loop since process part 2 anyway has faster dynamics than process part 1, so the secondary loop becomes fast enough. And in general the noise sensitive derivative term is a drawback. The primary controller is typically a PID controller or a PI controller.

In the secondary controller the P- and the D-term should not have reduced setpoint weights, cf. Section 2.7.1. Why?[2]

How do you *tune* the controllers of a cascade control? You can follow this procedure:

- First the secondary controller is tuned, with the primary controller in manual mode.
- Then the primary controller is tuned, the secondary controller in automatic mode.

Controller tuning can be made using a standard tuning method, e.g. the Ziegler-Nichols' closed loop method, cf. Section 4.4.

Example 9.4 ***Cascade control (simulation)***

In this example the following two control systems are simulated simultaneously (in parallel):

- A cascade control system consisting of two control loops.
- An ordinary single loop control system, which is simulated for comparison.

The process to be controlled is the same in both control systems, and they have the same setpoint, y_{SP}, and the same disturbance, v. The process consists of two partial processes in series, cf. Figure 9.7:

- Process P_1:
  $$y(s) = H_{P1}(s)y_2(s) \tag{9.16}$$
  where
  $$H_{P1}(s) = \frac{K}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\frac{s}{\omega_0} + 1} e^{-\tau s} \tag{9.17}$$
  with
  $$K = 1;\ \omega_0 = 0.2\text{rad/s};\ \zeta = 1;\ \tau = 1\text{s} \tag{9.18}$$

[2] Because attenuating or removing the time-varying setpoint (which is equal to the control signal produced by the primary controller) of the secondary loop will reduce the ability of the secondary loop to track these setpoint changes, causing slower tracking of the total control system.

- Process P_2:

$$y_2(s) = H_{P2}(s)u(s) + v(s) \tag{9.19}$$

where

$$H_{P2}(s) = \frac{K}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\frac{s}{\omega_0} + 1} e^{-\tau s} \tag{9.20}$$

with

$$K = 1;\ \omega_0 = 2\text{rad/s};\ \zeta = 1;\ \tau = 0.1\text{s} \tag{9.21}$$

Simply stated, process P_2 has ten times quicker dynamics than process P_1 has. The controllers have been tuned according to the Ziegler-Nichols' closed loop method with some fine-tuning to avoid too aggressive control action (increase of T_i from 0.69 to 1). The controller parameter settings are as follows:

- Cascade control system: Primary controller, C_1 (PID):

$$K_p = 2.1;\ T_i = 4.0;\ T_d = 1.0 \tag{9.22}$$

- Cascade control system: Secondary controller, C_2 (PI):

$$K_p = 1.5;\ T_i = 1.0;\ T_d = 0 \tag{9.23}$$

- Single loop control system: Controller, C (PID):

$$K_p = 1.9;\ T_i = 4.0;\ T_d = 1.0 \tag{9.24}$$

Figure 9.8 shows simulated responses with a step in the setpoint. IAE values for the two control systems are shown in Table 9.1. The IAE values

	Cascade control	**Single loop control**
Setpoint step	IAE = 17.18	IAE = 26.85
Disturbance step	IAE = 7.80	IAE = 84.13

Table 9.1: IAE values for cascade control system and for single loop control system

show that the setpoint tracking is better in the cascade control system, but not substantially better.

Figure 9.9 shows simulated responses with a step in the disturbance. The IAE values in Table 9.1 show that the disturbance compensation is much better in the cascade control system.

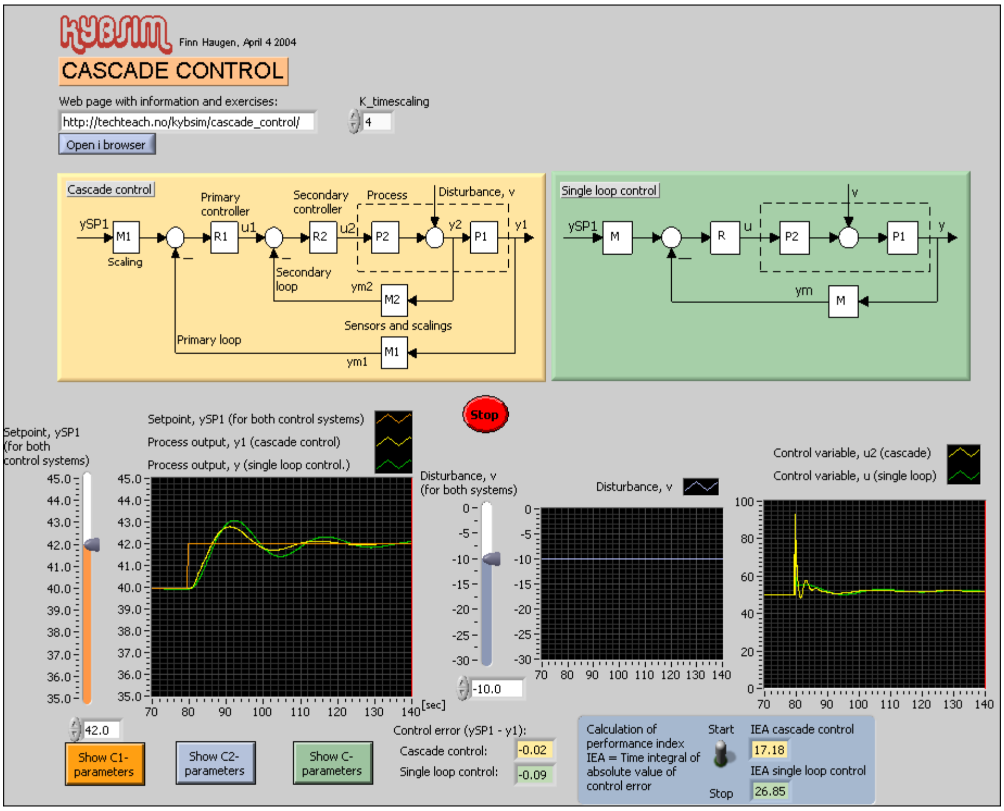

Figure 9.8: Example 9.4: Simulated responses with a step in the setpoint

Figure 9.9 shows that the control variable of the cascade control system works much more aggressively than in the single loop control system, which is due to the relatively quick secondary loop.

[End of Example 9.4]

Cascade control is frequently used in the industry. A few examples are described in the following.

Example 9.5 *Cascade control of the level in wood-chip tank*

Level control of a wood-chip tank has been a frequent example in this book. In the real level control system[3] cascade control is used, although not described in the previous examples. The primary loop performs level control. The secondary loop is a control loop for the mass flow on the

[3] at Södra Cell Tofte in Norway

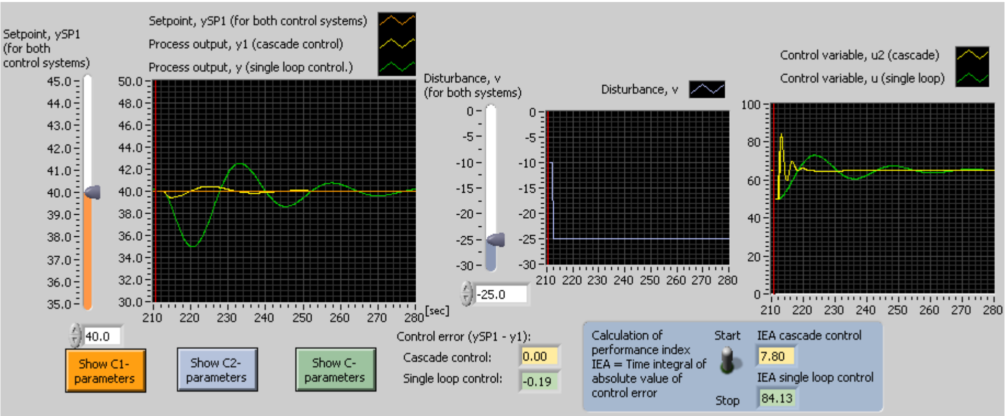

Figure 9.9: Example 9.4: Simulated responses with a step in the disturbance

conveyor belt, see Figure 9.10. The mass flow is measured by a flow sensor (which actually is based on a weight measurement of the belt with chip between two rollers). The purpose of the secondary loop is to give a quick compensation for disturbances in the chip flow due to variations in the chip consistency since the production is switched between spruce, pine and eucalyptus. In addition to this compensation the secondary loop gives a more linear or proportional relation between the control variable u and the mass flow w_s into the conveyor belt (at the flow sensor).

[End of Example 9.5]

Example 9.6 ***Cascade control of a heat exchanger***

Figure 9.11 shows a temperature control system for a heat exchanger. The control variable controls the opening of the hot water valve. The primary loop controls the product temperature. The secondary loop controls the heat flow to compensate for flow variations (disturbances). The valve with flow control system can be regarded a new valve with an approximate proportional relation between the control variable and the heat flow.

[End of Example 9.6]

There are many other examples of cascade control, e.g.:

- **DC-motor:**
  - Primary loop: Speed control based on measurement of the rotational speed using a tachometer as speed sensor.

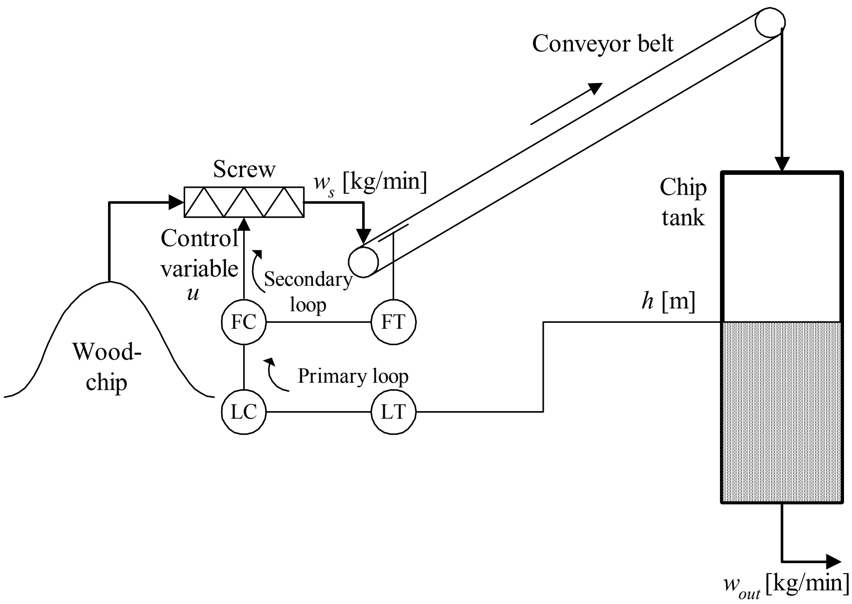

Figure 9.10: Example 9.5: Level control system of a wood-chip tank where the primary loop performs level control, and the secondary loop performs mass flow control. (FT = Flow Transmitter. FC = Flow Controller. LT = Level Transmitter. LC = Level Controller.)

  - Secondary loop: Control of armature current which compensates for nonlinearities of the motor, which in turn may give more linear speed control.

- **Hydraulic motor:**
  - Primary loop: Positional control of the cylinder
  - Secondary loop: Control of the servo valve position (the servo valve controls the direction of oil flow into the cylinder), which results in a more linear valve movement, which in turn gives a more precise control of the cylinder.

- **Control valve:**
  - The primary loop: Flow control of the liquid or the gas through the valve.
  - Secondary loop: Positional control of the valve stem, which gives a proportional valve movement, which in turn may give a more precise flow control. Such an internal positional control system is called positioner.

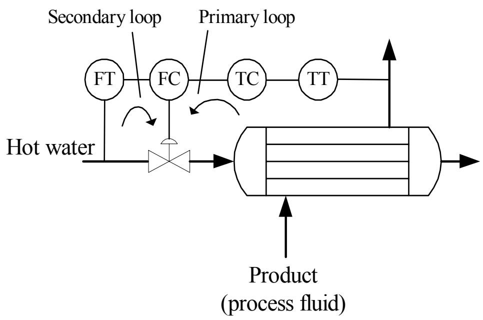

Figure 9.11: Example 9.6: Cascade control of the product temperature of a heat exchanger. (TC = Temperature Controller. TT = Temperature Transmitter. FC = Flow Controller. FT = Flow Transmitter.)

9.3 Ratio control and quality and product flow control

9.3.1 Ratio control

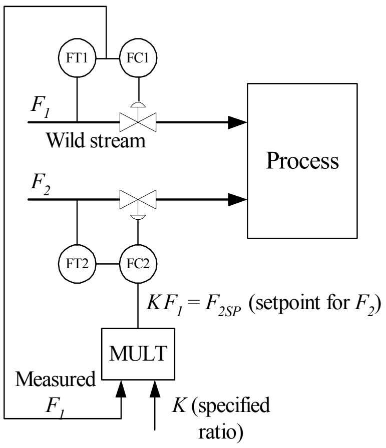

Figure 9.12: Ratio control

The purpose of *ratio control* is to control a mass flow, say F_2, so that the ratio between this flow and another flow, say F_1, is

$$F_2 = KF_1 \tag{9.25}$$

where K is a specified ratio which may have been calculated as an optimal ratio from a process model. One example is the calculation of the ratio between oil inflow and air inflow to a burner to obtain optimal operating condition for the burner. Another example is the nitric acid factory where ammonia and air must be fed to the reactor in a given ratio.

Figure 9.12 shows the structure of ratio control. The setpoint of the flow F_2 is calculated as K times the measured value of F_1, which is denoted the "wild stream". The figure shows a control loop of F_1. The setpoint of F_1 (the setpoint is not shown explicitly in the figure) can be calculated from a specified production rate of the process. The ratio control will then ensure the ratio between the flows as specified.

An alternative way to implement ratio control is to calculate the actual ratio as

$$K_{actual} = \frac{F_2}{F_1} \tag{9.26}$$

Then K_{actual} is used as a measurement signal to a ratio controller with the specified K as the setpoint and F_2 as the control variable, cf. Figure 9.13.

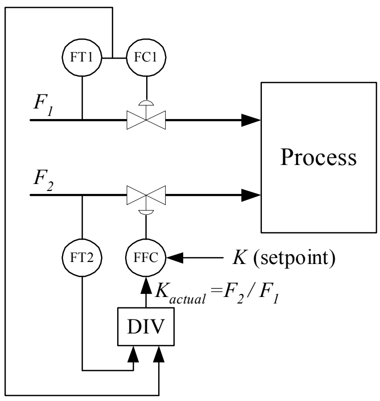

Figure 9.13: An alternative ratio control structure based on measurement of the actual ratio. (FFC = Flow Fraction Controller.)

Although this control structure is logical, it is a drawback that the loop gain, in which K_{actual} is a factor, is a function of the measurements of F_2 and F_1. Hence, this solution is not encouraged[16].

9.3.2 Quality and production rate control

Earlier in this section it was mentioned that the ratio K may origin from an analysis of optimal process operation, say from a specified product quality quantity, say Q_{SP}. Imagine however that there are disturbances so that key components in one of or in both flows F_1 or F_2 vary somewhat. Due to such disturbances it may well happen that the actual product quality is different from Q_{SP}. Such disturbances may also cause the actual product flow to differ from a flow setpoint. These problems can be solved by implementing

- a quality control loop based on feedback from measured quality Q to the ratio parameter K, and
- a product flow control loop based on feedback from measured flow F to one of the feed flows.

Figure 9.14 shows the resulting quality and production rate control system.

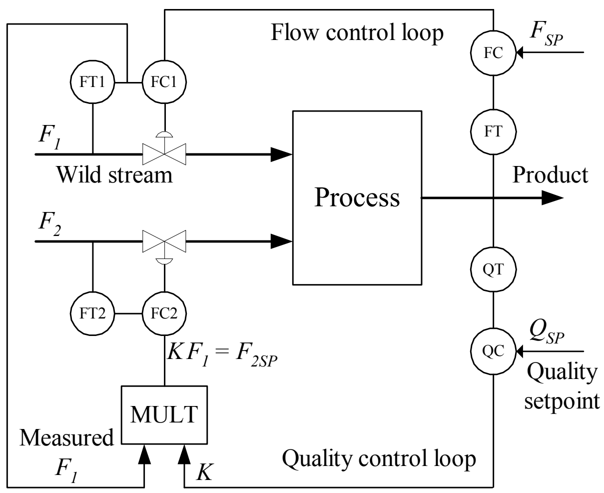

Figure 9.14: Control of quality and product flow. (QT = Quality Transmitter. QC = Quality Controller.)

9.4 Split-range control

In *split-range control* one controller controls two actuators in different ranges of the control signal span, which here is assumed to be 0 – 100%. See Figure 9.15. Figure 9.16 shows an example of split-range temperature control of a thermal process. Two valves are controlled – one for cooling and one for heating, as in a reactor. The temperature controller controls

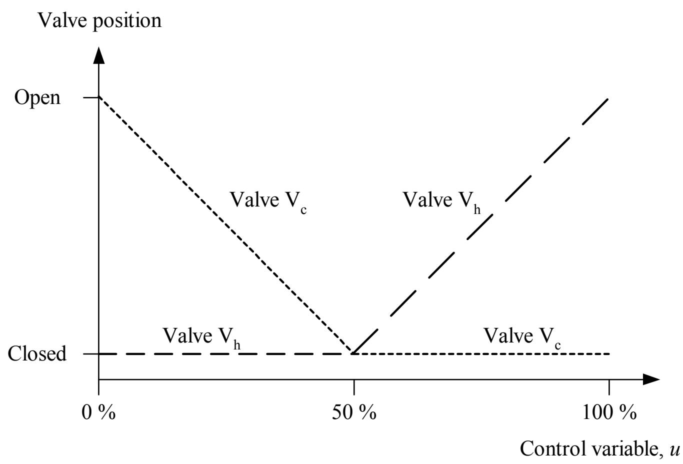

Figure 9.15: Split-range control of two valves

the cold water valve for control signals in the range 0–50%, and it controls the hot water valve for control signals in the range 50–100%, cf. Figure 9.15.

In Figure 9.15 it is indicated that one of the valves are open while the other is active. However in certain applications one valve can still be open while the other is active, see Figure 9.17. One application is pressure control of a process: When the pressure drop compensation is small (as when the process load is small), valve V_1 is *active* and valve V_2 is *closed.* And when the pressure drop compensation is is large (as when the process load is large), valve V_1 is *open* and valve V_2 *is still active.*

9.5 Control of product flow and mass balance in a plant

In the process industry products are created after treatment of the materials in a number of stages in series, which are typically unit processes as blending or heated tanks, buffer tanks, distillation columns, absorbers, reactors etc. The basic control requirements of such a production line are as follows:

- The mass flow of a key component must be controlled, that is, to follow a given production rate or flow setpoint.

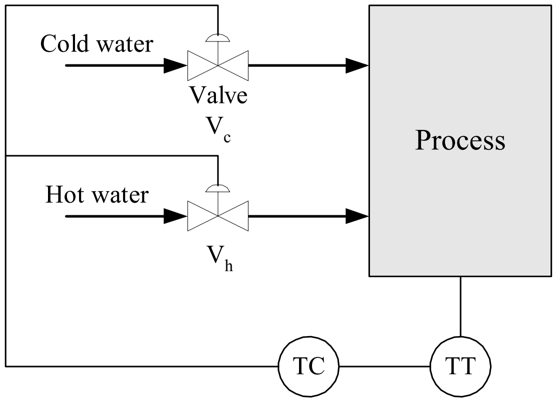

Figure 9.16: Split-range temperature control using two control valves

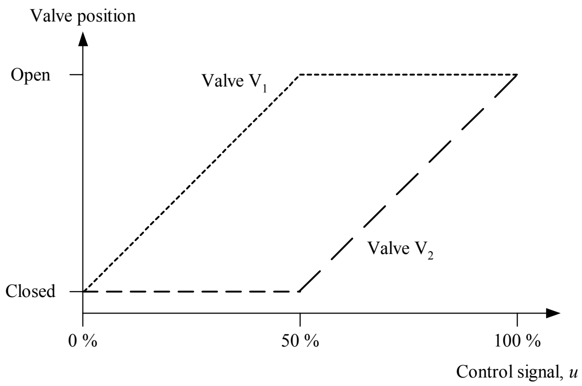

Figure 9.17: In split-range control one valve can be active while another valve is open simultaneuously.

- The mass balance in each process unit (tank etc.) must be maintained – otherwise e.g. the tank may go full or empty.

Figure 9.18 shows the principal control system structure to satisfy these requirements. (It is assumed that the mass is proportional to the level.) The position of the production flow control in the figure is just one example. It may be placed earlier (or later) in the line depending on where the key component(s) are added.

Note that the mass balance of an *upstream* tank (relative to the production flow control) is controlled by manipulating the mass *inflow* to the tank, while the mass balance of a *downstream* tank is controlled by manipulating the mass *outflow* to the tank.

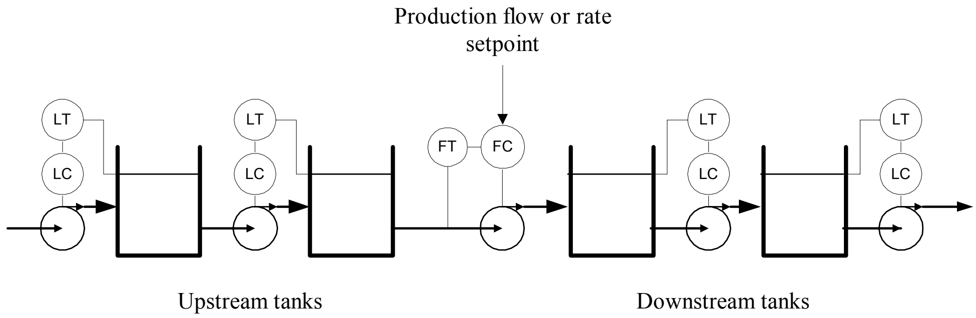

Figure 9.18: Control of a production line to maintain product flow (rate) and mass balances

In Figure 9.18 the mass balances are maintained using level control. If the tanks contains vapours, the mass balances are maintained using *pressure control*. Then pressure sensors (PT = Pressure Transmitter) takes the places of the level sensors (LT = Level Transmitters), and pressure controllers (PC = Pressure Controller) takes the place of level controllers (LC = Level Controller) in Figure 9.18.

Example 9.7 ***Control of production line***

Figure 9.19 shows the front panel of a simulator of a general production line. The level controllers are PI controllers which are tuned so that the control loops get proper speed and stability (the parameters may be calculated as explained in Chapter 7.2.2). The production flow F is here controlled using a PI controller. Figure 9.19 shows how the level control loops maintain the mass balances (in steady-state) by compensating for a disturbance which is here caused by a change of the production flow. Note that controller LC2 must have negative gain (i.e. direct action, cf. Section 2.6.8) – why?[4]

[End of Example 9.7]

[4] Because the process has negative gain, as an increase of the control signal gives a reduction of the level/level measurement.

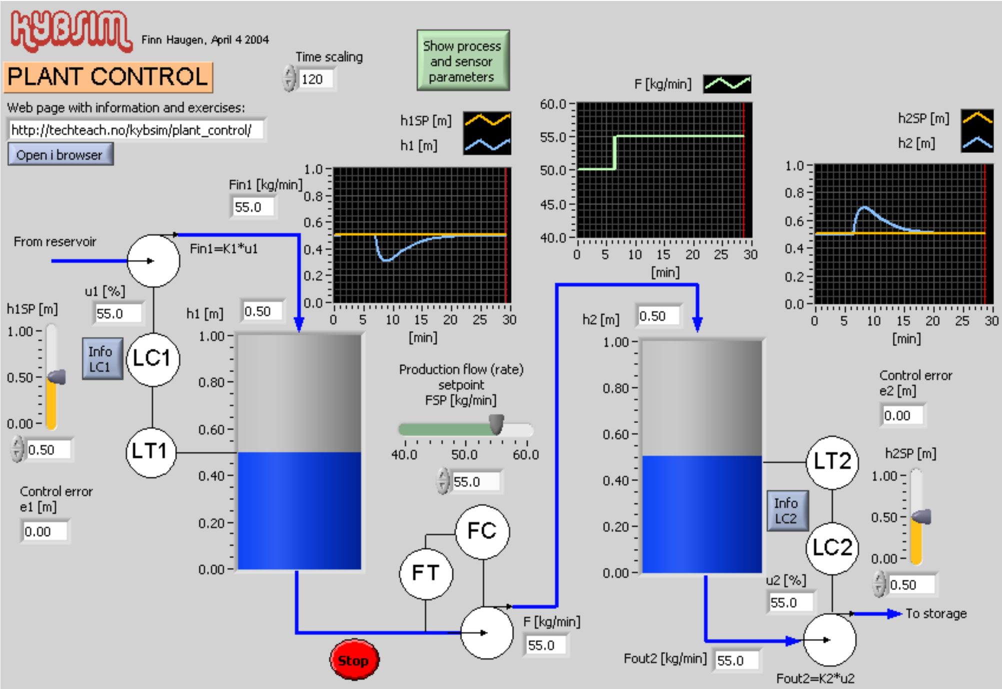

Figure 9.19: Example 9.7: The level control loops maintain the mass balances (in steady-state).

9.6 Multivariable control

9.6.1 Introduction

Multivariable processes has more than one input variables or ore than one output variables. Here are a few examples of multivariable processes:

- A heated liquid tank where both the level and the temperature shall be controlled.
- A distillation column where the top and bottom concentration shall be controlled.
- A robot manipulator where the positions of the manipulators (arms) shall be controlled.
- A chemical reactor where the concentration and the temperature shall be controlled.