7.2 Controller tuning from specified characteristic polynomial

7.2.1 Introduction

The subsequent sections explain controller tuning based on specifications of the characteristic polynomial of the control system. Using this method you can shape the dynamic properties of the control system quite freely. However, the method is in practice applicable only to processes of low order due to the mathematical operations involved, and here only integrator processes and first order (time constant) process will be considered.

If the process order is high, or if the process contains time delay, you should consider using the Ziegler-Nichols' tuning methods, cf. Chapter 4 (Ziegler-Nichols' tuning methods actually can not be used for integrators or first order processes since the parameters needed in the methods, as the ultimate gain, can not be found or is infinitely large for these processes).

For all the processes that we soon will encounter (integrator and first order system), Skogestad's method, cf. Section 7.5, can be used. Although this tuning method certainly works fine, the method is based on some model approximations. In some cases it is useful to be able to perform an exact controller design. One important example is the level controller design for a liquid tank, cf. Example 7.2.

7.2.2 Tuning a controller for an integrator process

The process transfer function is

$$H_p(s) = \frac{K}{s} \tag{7.9}$$

and the disturbance transfer function is

$$H_{vm}(s) = \frac{K_{vm}}{s} \tag{7.10}$$

One example of such a process is a liquid tank where the level h is to be controlled by controlling the outflow w_{out} from the tank. The transfer function from w_{out} to level measurement h_m is on the form (7.9). (This example is described in detail in Example 7.1 (page 194).)

We will use a PI controller (the derivative term in the PID controller serves no purpose for this process), which has transfer function

$$H_c(s) = K_p \frac{T_i s + 1}{T_i s} \tag{7.11}$$

The controller parameters K_p and T_i will be calculated from a specified *bandwidth*, which represents the speed of the control system. In addition we must require that the control system has acceptable *stability*. We start by finding the tracking transfer function $T(s)$, which is given by (7.8) where the loop transfer function is

$$L(s) = H_c(s)H_p(s) = K_p \frac{T_i s + 1}{T_i s} \cdot \frac{K}{s} \tag{7.12}$$

From (7.8) we get

$$T(s) = \frac{L(s)}{1 + L(s)} = \frac{K_p K \left(s + \frac{1}{T_i}\right)}{\underbrace{s^2 + K_p K s + \frac{K_p K}{T_i}}_{c(s)}} \tag{7.13}$$

where $c(s)$ is the characteristic polynomial of the control system. We write it as a standard second order polynomial:

$$c(s) = s^2 + K_p K s + \frac{K_p K}{T_i} = s^2 + 2\zeta\omega_0 s + {\omega_0}^2 \tag{7.14}$$

where ω_0 is the undamped resonance frequency and ζ is the relative damping factor [7]. Comparison of coefficients between the two polynomials in (7.14) gives the following identities:

$$K_p K \equiv 2\zeta\omega_0 \text{ and } \frac{K_p K}{T_i} \equiv {\omega_0}^2 \tag{7.15}$$

Solving for K_p and T_i gives the following formulas for the controller parameters:

$$K_p = \frac{2\zeta\omega_0}{K} \tag{7.16}$$

$$T_i = \frac{2\zeta}{\omega_0} \tag{7.17}$$

Using (7.16) and (7.17), $T(s)$ can be written as

$$T(s) = \frac{K_p K \left(s + \frac{1}{T_i}\right)}{s^2 + K_p K s + \frac{K_p K}{T_i}} = \frac{2\zeta\omega_0 s + {\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2} \tag{7.18}$$

ω_0 can be interpreted as the bandwidth of the tracking function (7.18). A rough estimate of the response time[1] of the control system is

$$T_r \approx \frac{1}{\omega_0} \tag{7.19}$$

A reasonable choice of ζ is

$$\zeta = 0.5 \tag{7.20}$$

which gives step responses with well damped oscillations. If larger damping of the time responses is desired, ζ can be given a larger value (closer to 1).

Example 7.1 ***PI control of an integrator process***

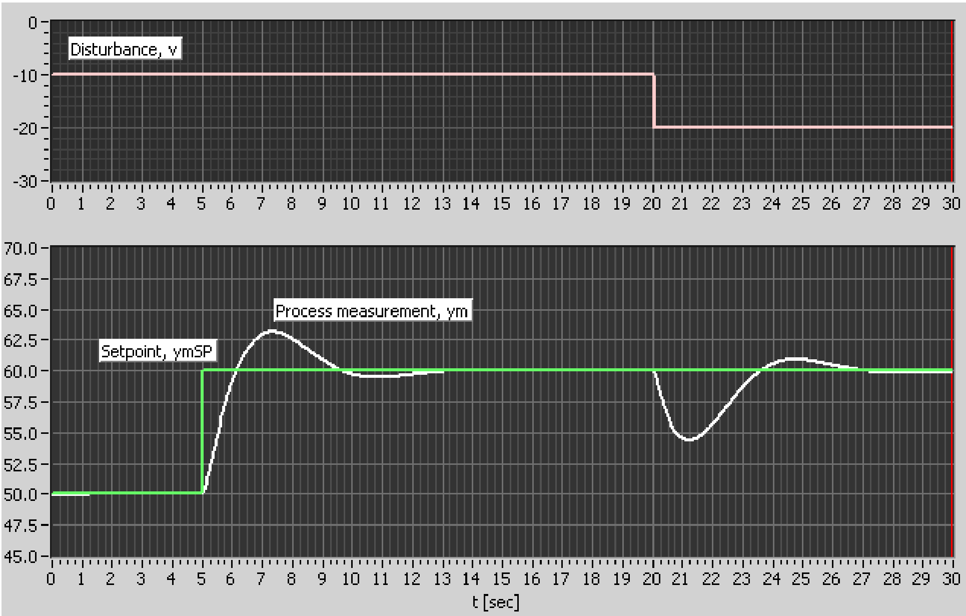

Figure 7.3: Example 7.1: Simulated responses in the control system

Assume that $K = 1$ and $K_{vm} = 1$ in (7.9) and (7.10). We specify $\omega_0 = 1$ and $\zeta = 0.5$. (7.16) and (7.17) gives

$$K_p = 1; \, T_i = 1 \tag{7.21}$$

Figure 7.3 shows simulated responses in a control system with transfer functions (7.9) and (7.10). There is a setpoint step and a disturbance step. The simulations indicates that the stability of the control system is

[1] The response time can be regarded as an approximate time constant.

acceptable. The response time is read off as $T_r \approx 0.9$s which is quite similar to the estimate $T_r = 1/\omega_0 = 1/1 = 1$s according to (7.19).

[End of Example 7.1]

Tuning the controller for *sluggish* control

The aim of controller tuning is not always fast control, but in stead sluggish control! This is the case for a level controlled liquid tank in a process line. The tank is an integrator, dynamically. The level control system ensures the mass balance. In addition the control system behaves like a lowpass filter between the (free) inflow w_{in} and the outflow w_{out}. To obtain enough attenuation of inflow variations through the system, the level control system must be sluggish! Example 7.2 goes into the details.

Example 7.2 ***Level control of buffer tank***

Figure 7.4 shows the front panel of a simulator for buffer tank with level control system.[2] (The simulated responses are explained later in this example.)The control system has two aims:

- To keep the level on or close to a level setpoint.
- To attenuate variations in the outflow so that it becomes smoother than the inflow.

We need a mathematical process model: Mass balance is

$$\rho A\dot{h} = w_{in} - \underbrace{w_{out}}_{K_u u} \tag{7.22}$$

Laplace transformation of (7.22) is

$$\rho A h(s) = w_{in}(s) - K_u u(s) \tag{7.23}$$

Solving for $h(s)$ gives the following transfer function model:

$$h(s) = \frac{1}{\rho A s} w_{in}(s) - \frac{K_u}{\rho A s} u(s) \tag{7.24}$$

[2] The system may be in e.g. a production line in a factory.

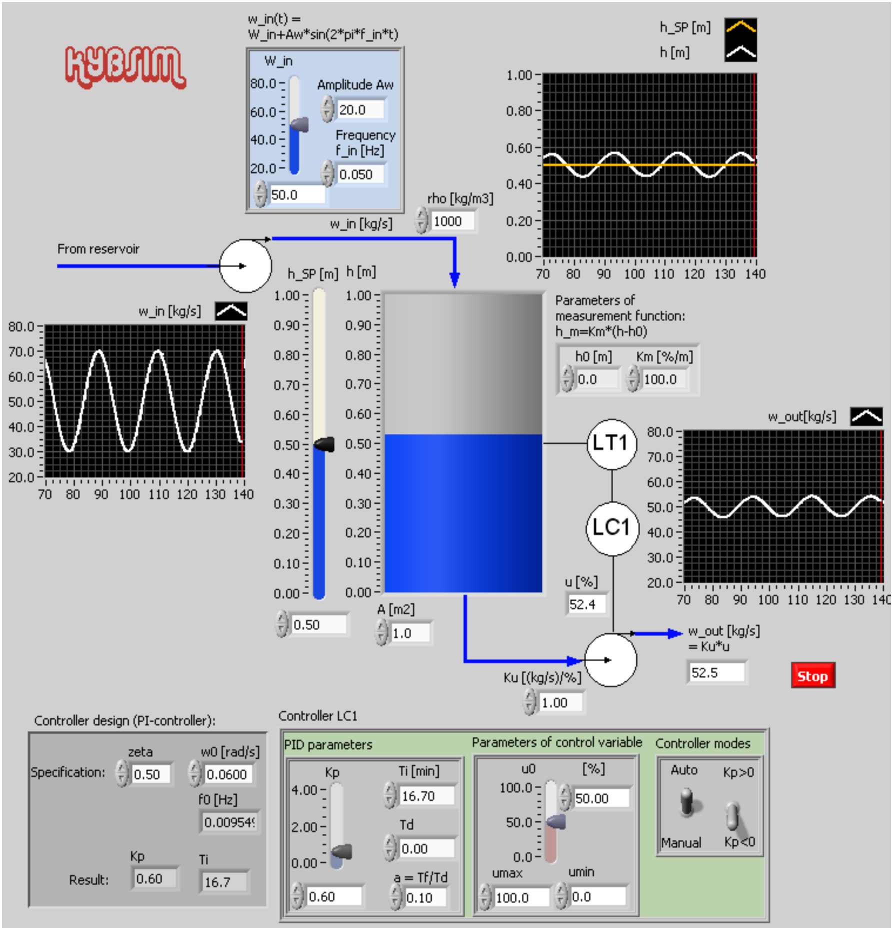

Figure 7.4: Example 7.2: Front panel of simulator for level control system

The transfer function from level h to level measurement h_m is

$$h_m(s) = K_m h(s) \tag{7.25}$$

Combining (7.24) and (7.25) gives the following model:

$$h_m(s) = \frac{K_m}{\rho A s} w_{in}(s) - \frac{K_m K_u}{\rho A s} u(s) \tag{7.26}$$

The transfer function from u to h_m is

$$H_p(s) = \frac{h_m(s)}{u(s)} = -\frac{K_u}{\rho A s} = -\frac{K}{s} \tag{7.27}$$

where

$$K = \frac{K_u K_m}{\rho A} \tag{7.28}$$

is the process gain. Figure 7.5 shows a block diagram of the level control system.

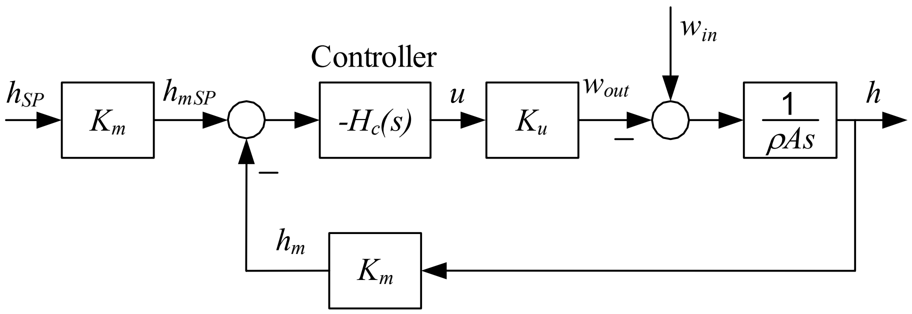

Figure 7.5: Example 7.2: Block diagram of the level control system

The level controller is a PI controller. The integral term ensures zero static control error. The transfer function of the PI controller is

$$H_c(s) = K_p \frac{T_i s + 1}{T_i s} \tag{7.29}$$

What is the reason for the negative sign ahead of the controller transfer function $H_c(s)$ in the block diagram in Figure (7.5)? The negative sign means that the controller in effect has negative gain. Negative controller gain is here necessary since the process gain is negative, cf. Section 2.6.8.

The tracking transfer function of the control system is given by (7.18), which is repeated here:

$$\frac{h_m(s)}{h_{m_{SP}}(s)} = T(s) \tag{7.30}$$

$$= \frac{L(s)}{1 + L(s)} \tag{7.31}$$

$$= \frac{H_c(s)H_p(s)}{1 + H_c(s)H_p(s)} \tag{7.32}$$

$$= \frac{K_p K \left(s + \frac{1}{T_i}\right)}{s^2 + K_p K s + \frac{K_p K}{T_i}} \tag{7.33}$$

$$= \frac{2\zeta\omega_0 s + {\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2} \tag{7.34}$$

Once ζ and ω_0 is specified, the controller parameters are given by (7.16) and (7.17). Below we will specify ζ and ω_0 from a specification to the attenuation of the mass flow through the tank.

The relation between the inflow w_{in} and the outflow w_{ut} can be expressed by the transfer function from w_{in} to w_{out}. From the block diagram in Figure 7.5 we see that the relation between w_{in} and w_{out} is identical to the relation between the level setpoint h_{SP} and the level h, which implies that the transfer function from w_{in} to w_{out} is the same as the tracking transfer function! Thus,

$$\frac{w_{out}(s)}{w_{in}(s)} = \frac{h(s)}{h_m(s)} = T(s) \tag{7.35}$$

$$= \frac{K_pK\left(s+\frac{1}{T_i}\right)}{s^2+K_pKs+\frac{K_pK}{T_i}} \tag{7.36}$$

$$= \frac{2\zeta\omega_0 s+{\omega_0}^2}{s^2+2\zeta\omega_0 s+{\omega_0}^2} \tag{7.37}$$

By giving values to ζ and ω_0 we determine the dynamic properties of the controlled tank. Let us set

$$\zeta = 0.5 \tag{7.38}$$

What about ω_0? It can roughly be regarded as the bandwidth of the lowpass filter (7.35). A Bode plot of the amplitude function $|T(j\omega)|$ gives a good picture of the filtering properties, see Figure 7.6 which shows $|T(j\omega)|$ with controller parameters as calculated later in this example. In the figure the frequency unit is Hz. The relation between a frequency f_1 in Hz and the corresponding frequency ω_1 in rad/s is

$$2\pi f_1 = \omega_1 \tag{7.39}$$

Let us specify that a frequency component in w_{in} of frequency $f_{in} = 0.05$Hz is attenuated by a factor of 5 – or in other words: amplified by factor 0.2 which is approximately -14dB. This means that the amplitude gain of T at this frequency must be

$$|T(s)|_{s=j2\pi f_{in}} \tag{7.40}$$

$$= |T(j2\pi f_{in})| \tag{7.41}$$

$$= \left| \left. \frac{2\zeta\omega_0 s+{\omega_0}^2}{s^2+2\zeta\omega_0 s+{\omega_0}^2} \right|_{s=j2\pi f_{in}=j2\pi\cdot 0.05} \right| = 0.2 = -14\text{dB} \tag{7.42}$$

Here we use (7.38). In principle we can now solve (7.42) for ω_0 (to be used in (7.16) and (7.17) for calculating the PI parameters). Although it is possible to solve (7.42) for ω_0, it is a bit difficult operation. If we have a computer tool for plotting Bode diagrams, it is easier to iterate on plotting $|T|$ for varying ω_0 until $|T| = 0.2$. The result is

$$\omega_0 = 0.06\text{rad/s} \mathrel{\hat{=}} 0.0095\text{Hz} \tag{7.43}$$

Using (7.38) and (7.43) in (7.16) and (7.17) gives

$$K_p = 0.60;\ T_i = 16.7\text{s} \tag{7.44}$$

Figure 7.6 shows a Bode plot of $|T(j\omega)|$ with the controller parameters (7.44). We read off $|T| = -14.0\text{dB} = 0.20$.

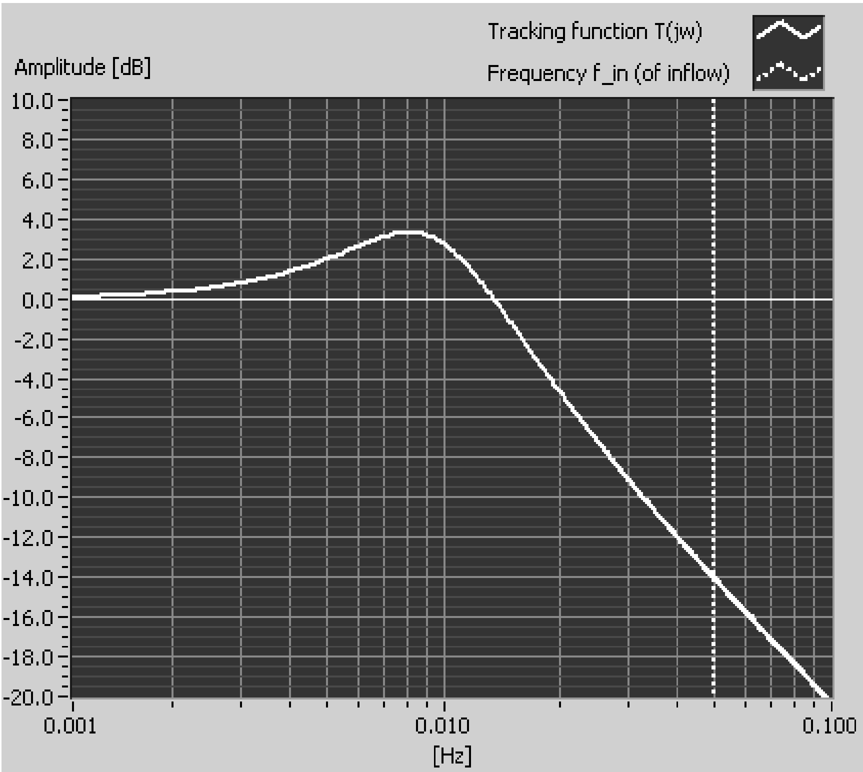

Figure 7.6: Example 7.2: Bode plot of $|T(\omega)|$ with the calculated PI parameters

Figure 7.4 shows simulated responses in the control system with parameter values defined above. An accurate reading from the simulations shows that the amplitude of w_{out} is 4.0kg/s (in steady-state). The amplitude of w_{in} is 20kg/s. Thus, the amplitude ratio is $4.0/20 = 0.20 = -14.0\text{dB}$, which is in accordance with the Bode plot, see Figure 7.6.

[End of Example 7.2]