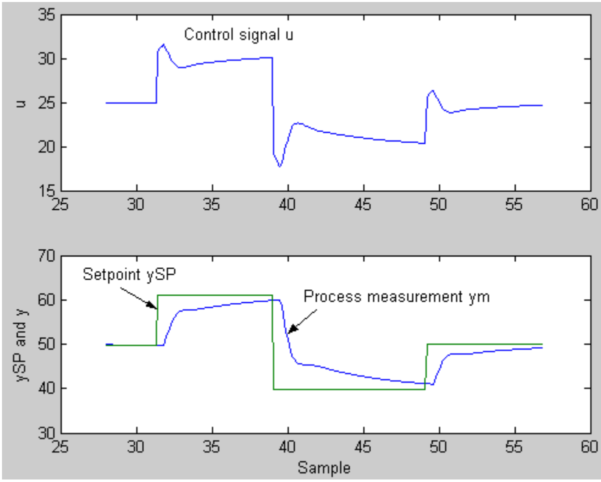

Figure 4.19: Example 4.6: Control signal u and process measurement y_m used for estimation of process model

parameter values (4.48). The re-tuning has clearly given an improvement of the quickness of the control loop, and the stability of the control loop is satisfactory.

[End of Example 4.6]

4.9 PID tuning when process dynamics varies

4.9.1 Introduction

A well tuned PID controller has parameters which are adapted to the dynamic properties to the process, so that the control system becomes fast and stable. If the process dynamic properties varies without re-tuning the controller, the control system

- gets *reduced stability* or
- becomes *more sluggish*.

Problems with variable process dynamics can be solved as follows:

- **The controller is tuned in the most critical operation point**, so that when the process operates in a different operation point, the

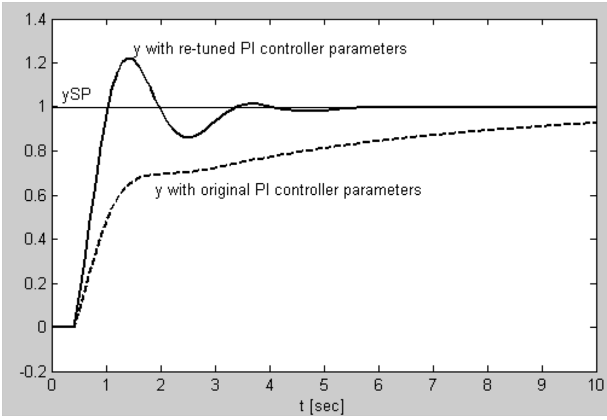

Figure 4.20: Example 4.6: Response in process output for control system with original and re-tuned PI controller parameters

stability of the control system is just better — at least the stability is not reduced. However, if the stability is too good the tracking quickness is reduced, giving more sluggish control.

- **The controller parameters are varied in the "opposite" direction of the variations of the process dynamics**, so that the performance of the control system is maintained, independent of the operation point. Two ways to vary the controller parameters are:
  - *PID controller with gain scheduling.* This is described in detail in Section 4.9.2.
  - *Model-based adaptive controller.* This is described briefly in Section 4.9.4.

Commercial control equipment is available with options for gain scheduling and/or adaptive control.

4.9.2 Gain scheduling PID controller

Figure 4.21 shows the structure of a control system for a process which may have varying dynamic properties, for example a varying gain. The *Gain scheduling variable* GS is some measured process variable which at every instant of time expresses or represents the dynamic properties of the process. As you will see in Example 4.7, GS may be the mass flow through a liquid tank.

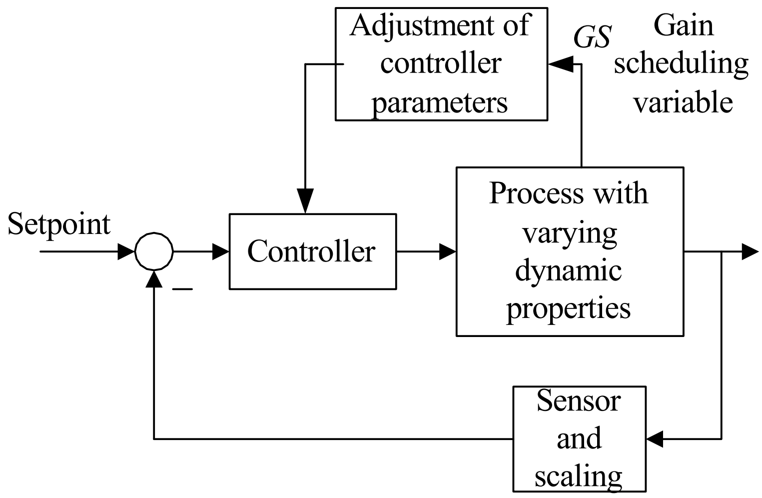

Figure 4.21: Control system for a process having varying dynamic properties. The GS variable expresses or represents the dynamic properties of the process.

Assume that proper values of the PID parameters K_p, T_i and T_d are found using for example Ziegler-Nichols' closed loop method for a set of values of the GS variable. These PID parameter values can be stored in a parameter table – the gain schedule – as shown in Table 4.3. From this table proper PID parameters are given as functions of the gain scheduling variable, GS.

GS	K_p	T_i	T_d
P_1	K_{p_1}	T_{i_1}	T_{d_1}
P_2	K_{p2}	T_{i_2}	T_{d_2}
P_3	K_{p_3}	T_{i_3}	T_{d_3}

Table 4.3: Gain schedule or parameter table of PID controller parameters.

There are several ways to express the PID parameters as functions of the GS variable:

- **Piecewise constant controller parameters**: An interval is defined around each GS value in the parameter table. The controller parameters are kept constant as long as the GS value is within the interval. This is a simple solution, but is seems nonetheless to be the most common solution in commercial controllers.

  When the GS variable changes from one interval to another, the controller parameters are changed abruptly, see Figure 4.22 which illustrates this for K_p, but the situation is the same for T_i and T_d. In

Figure 4.22 it is assumed that GS values toward the left are critical with respect to the stability of the control system. In other words: It is assumed that it is safe to keep K_p constant and equal to the K_p value in the left part of the the interval.

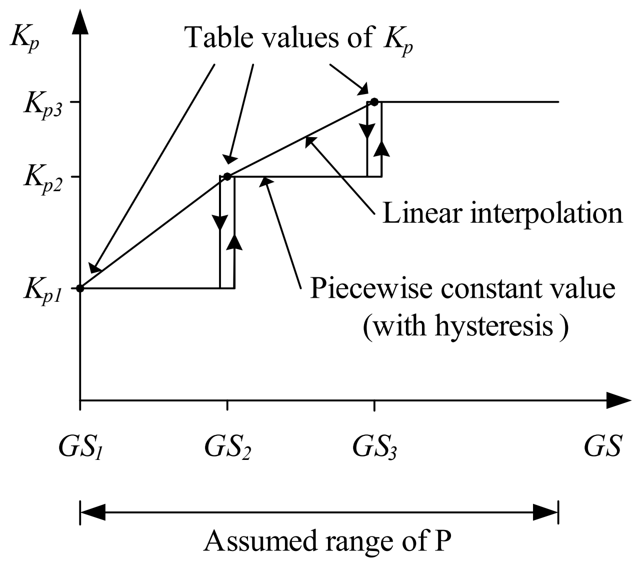

Figure 4.22: Two different ways to interpolate in a PID parameter table: Using piecewise constant values and linear interpolation

Using this solution there will be a disturbance in the form of a step in the control variable when the GS variable shifts from one interval to a another, but this disturbance is probably of negligible practical importance for the process output variable. Noise in the GS variable may cause frequent changes of the PID parameters. This can be prevented by using a hysteresis, as shown in Figure 4.22.

- **Piecewise interpolation**, which means that a linear function is found relating the controller parameter (output variable) and the GS variable (input variable) between to adjacent sets of data in the table. The linear function is on the form

$$K_p = a \cdot GS + b \tag{4.49}$$

where a and b are found from the two corresponding data sets:

$$K_{p_1} = a \cdot GS_1 + b \tag{4.50}$$

$$K_{p_2} = a \cdot GS_2 + b \tag{4.51}$$

(Similar equations applies to the T_i parameter and the T_d parameter.) (4.50) and (4.51) constitute a set of two equations with two unknown variables, a and b.[8]

- Other interpolations may be used, too, for example a polynomial function fitted exactly to the data or fitted using the least squares method.

Example 4.7 ***Gain schedule based PID temperature control at variable mass flow***

Figure 4.25 shows the front panel of a simulator for a temperature control system for a liquid tank with variable mass flow, w, through the tank. The control variable u controls the power to heating element. The temperature T is measured by a sensor which is placed some distance away from the heating element. There is a time delay from the control variable to measurement due to imperfect blending in the tank.

The process dynamics We will initially, both in simulations and from analytical expressions, that the dynamic properties of the process *varies with the mass flow w*. The response in the temperature T is simulated for the following two open loop cases (i.e., not feedback control):

- A step in u of amplitude 10% from 31.5% to 41.5% at mass flow $w = 12$kg/min, which in this context is a relatively *small* value, see Figure 4.23.
- A step in u of amplitude 10%, from 63.0 % to 73.0 % at $w = 24$kg/min, which in this context is a relatively *large* value, see Figure 4.24.

The simulations show that the following happens when the mass flow w is reduced (from 24 to 12kg/min): The gain process K is larger, the time constant T_t is larger, and the time delay τ is larger. (These terms assumes that system is a first order system with time delay. The simulator is based on such a model. The model is described below.)

Let us see if the way the process dynamics seems to depend on the mass flow w as seen from the simulations, can be confirmed from a

[8] The solution is left to you.

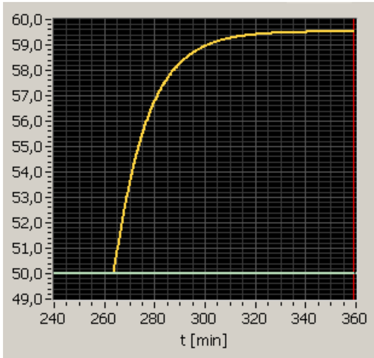

Figure 4.23: Response in temperature T after a step in u of amplitude 10% from 31.5% to 41.5% at the small mass flow $w = 12$kg/min

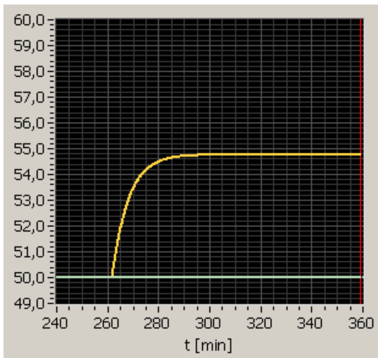

Figure 4.24: Response in temperature T after a step in u of amplitude 10% from 63.0% to 73.0% at the large mass flow $w = 24$kg/min

mathematical process model.[9] Assuming perfect stirring in the tank to have homogeneous conditions in the tank, we can set up the following energy balance for the liquid in the tank:

$$c\rho V\dot{T}_1(t) = K_P u(t) + cw\left[T_{in}(t) - T_t(t)\right] \tag{4.52}$$

where T_1 [K] is the liquid temperature in the tank, T_{in} [K] is the inlet temperature, c [J/(kg K)] is the specific heat capacity, V [m^3] is the liquid volume, ρ [kg/m^3] is the density, w [kg/s] is the mass flow (same out as in), K_P [W/%] is the gain of the power amplifier, u [%] is the control variable, $c\rho V T_1$ is (the temperature dependent) energy in the tank. It is assumed that the tank is isolated, that is, there is no heat transfer through

[9] Well, it would be strange if not. After all, we will be analyzing the same model as used in the simulator.

the walls to the environment. To make the model a little more realistic, we will include a time delay τ [s] to represent inhomogeneous conditions in the tank. Let us for simplicity assume that the time delay is inversely proportional to the mass flow. Thus, the temperature T at the sensor is

$$T(t) = T_1(t - \underbrace{\frac{K_\tau}{w}}_{\tau}) \tag{4.53}$$

where τ is the time delay and K_τ is a constant. Let us study the transfer function from u to T. Taking the Laplace transform of (4.52) gives

$$c\rho V \left[sT_1(s) - T_{1_0} \right] = K_P u(s) + cw \left[T_{in}(s) - T_t(s) \right] \tag{4.54}$$

where T_{1_0} is the initial value of T. Rearranging (4.54) yields the following model

$$T_1(s) = \frac{\frac{\rho V}{w}}{\frac{\rho V}{w}s + 1} T_{1_0} + \frac{\frac{K_P}{cw}}{\frac{\rho V}{w}s + 1} u(s) + \frac{1}{\frac{\rho V}{w}s + 1} T_{in}(s) \tag{4.55}$$

Taking the Laplace transform of (4.53) gives

$$T(s) = e^{-\frac{K_\tau}{w}s} T_1(s) \tag{4.56}$$

Substituting $T_1(s)$ in (4.56) by $T_1(s)$ from (4.55) yields the following transfer function $H_u(s)$ from u to T:

$$T(s) = \frac{\overbrace{\frac{K_P}{cw}}^{K}}{\underbrace{\frac{\rho V}{w}}_{T_t} s + 1} e^{-\overbrace{\frac{K_\tau}{w}}^{\tau} s} u(s) \tag{4.57}$$

$$= \underbrace{\frac{K}{T_t s + 1} e^{-\tau s}}_{H_u(s)} u(s) \tag{4.58}$$

Thus,

$$K = \frac{K_P}{cw} \tag{4.59}$$

$$T_t = \frac{\rho V}{w} \tag{4.60}$$

$$\tau = \frac{K_\tau}{w} \tag{4.61}$$

This confirms the observations in the simulations: Reduced mass flow w implies larger process gain, larger time constant, and larger time delay.

Heat exchangers and blending tanks in a process line where the production rate or mass flow varies, have similar dynamic properties as the tank in this example.

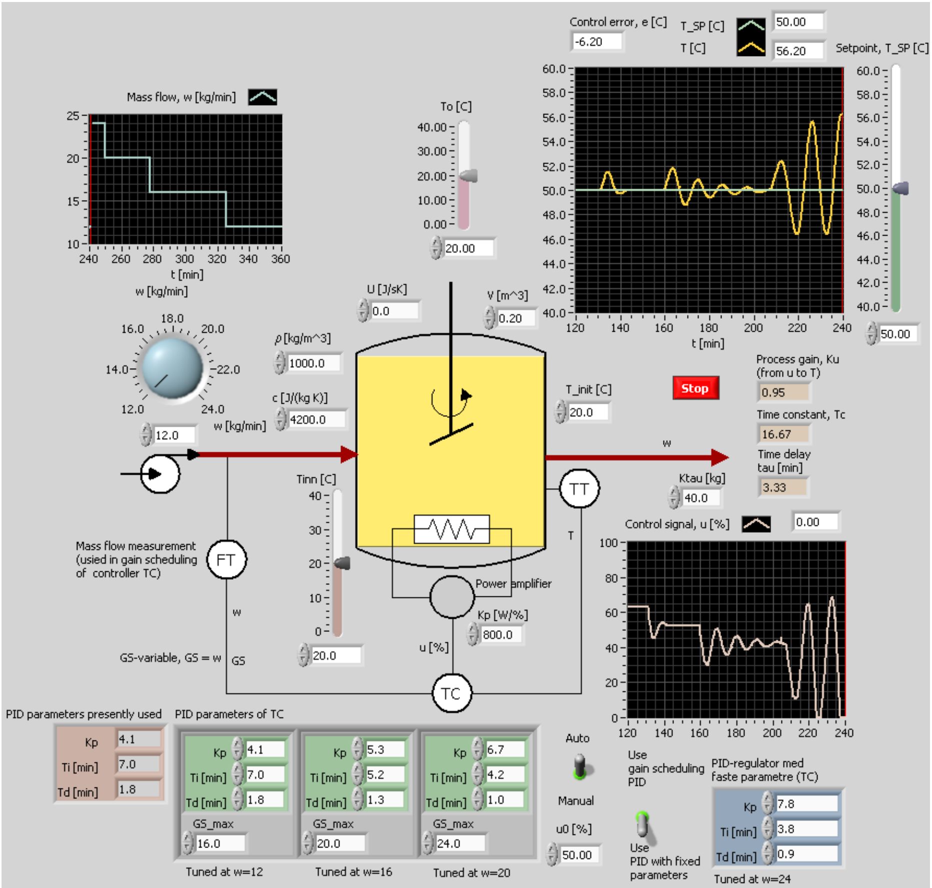

Figure 4.25: Example 4.7: Simulation of temperature control system with PID controller with fixed parameters tuned at maximum mass flow, which is $w = 24\text{kg/min}$

Control without gain scheduling (with fixed parameters) Let us look at temperature control of the tank. The mass flow w varies. In which operating point should the controller be tuned if we want to be sure that the stability of the control system is not reduced when w varies? In general the stability of a control loop is reduced if the gain increases and/or if the time delay of the loop increases. (4.59) and (4.61) show how the gain and time delay depends on the mass flow w. According to (4.59) and (4.61) the PID controller should be tuned at minimal w. If we do the opposite, that is, tune the controller at the maximum w, the control system may actually become unstable if w decreases.

Let us see if a simulation confirms the above analysis. Figure 4.25 shows a temperature control system. The PID controller is in the example tuned with the Ziegler-Nichols' closed loop method for a the maximum w value, which here is assumed 24kg/min. The PID parameters are

$$K_p = 7.8;\ T_i = 3.8\text{min};\ T_d = 0.9\text{min} \tag{4.62}$$

Figure 4.25 shows what happens at a stepwise reduction of w: The stability becomes worse, and the control system becomes *unstable* at the minimal w value, which is 12kg/min.

Instead of using the PID parameters tuned at maximum w value, we can tune the PID controller at minimum w value, which is 12kg/min. The parameters are then

$$K_p = 4.1;\ T_i = 7.0\text{min};\ T_d = 1.8\text{min} \tag{4.63}$$

The control system will now be stable for all w values, but the system behaves sluggish at large w values. (Responses for this case is however not shown here.)

Control with gain scheduling Let us see if gain scheduling maintains the stability for varying mass flow w. The PID parameters will be adjusted as a function of a measurement of w since the process dynamics varies with w. Thus, w is the gain scheduling variable, GS:

$$GS = w \tag{4.64}$$

A gain schedule consisting of three PID parameter value sets will be used. Each set is tuned using the Ziegler-Nichols' closed loop method at the following GS or w values: 12, 16 and 20kg/min. These three PID parameter sets are shown down to the left in Figure 4.25. The PID parameters are held piecewise constant in the GS intervals. In each interval, the PID parameters are held fixed for an increasing $GS = w$ value, cf. Figure 4.22.[10] Figure 4.26 shows the response in the temperature for decreasing values of w. The simulation shows that the *stability of the control system is maintained even if w decreases.*

[End of Example 4.7]

4.9.3 Adjusting PID parameters from process model

In Section 4.9.2 the adjustment of the PID parameters was based on interpolating between PID parameter values in a parameter table. However, a table with interpolation is not the only way the adjustment can

[10] The simulator uses the inbuilt gain schedule in LabVIEW's PID Control Toolkit.

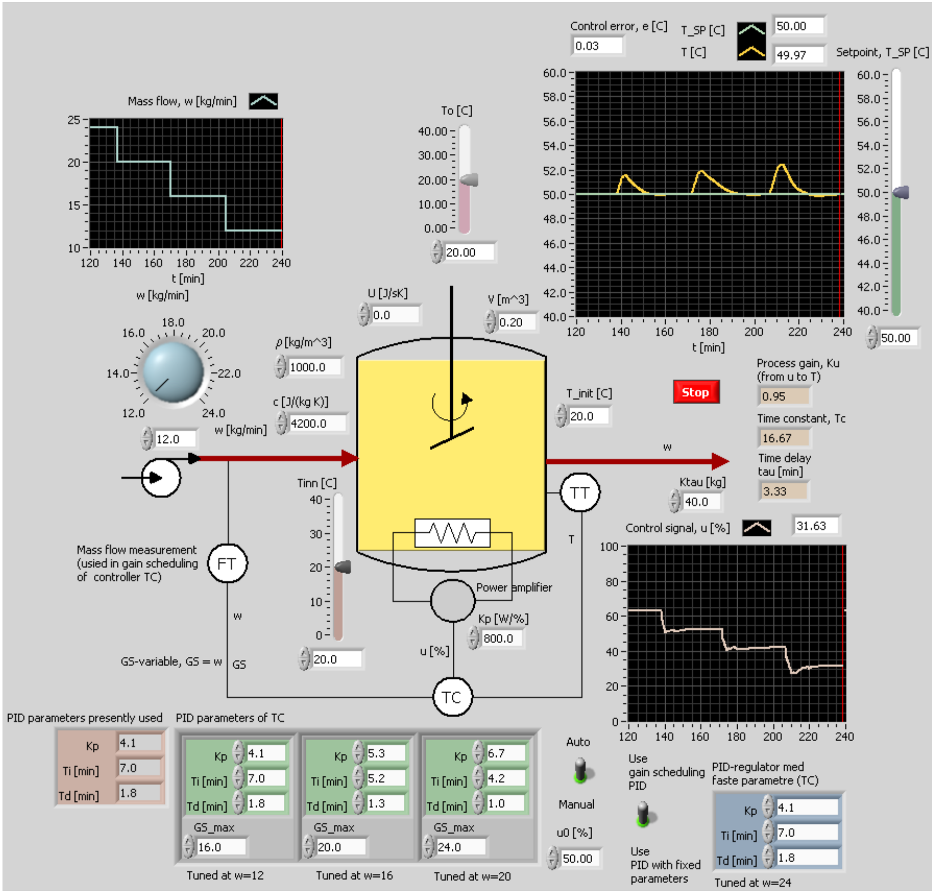

Figure 4.26: Example 4.7: Simulation of temperature control system with a gain schedule based PID controller

be implemented. By studying the process model we may find a function for parameter adjustment without having to make tuning in a number of operating points. Assume as an example that the process gain K is a function of a process variable P:

$$K = f_K(P) \tag{4.65}$$

In many control loops the stability of the loop is maintained if the loop gain K_L, which is the product of the gain of each subsystem in the loop, is constant, say K_{L_0}. In other words, the stability is maintained if

$$K_L = K_p K K_s = K_p f_K(P) K_s = K_{L_0} \tag{4.66}$$

where K_p is the controller gain (of a P or PI or PID controller) and K_m is the measurement gain (including a scaling function). For a given P value, say P_1,

$$K_{p_1} f_K(P_1) K_s = K_{L_0} \tag{4.67}$$

where K_{p_1} is assumes to be a proper K_p value (found using some tuning method) when $P = P_1$. By dividing (4.66) by (4.67) we get

$$\frac{K_p f_K(P) K_s}{K_{p_1} f_K(P_1) K_s} = \frac{K_{L_0}}{K_{L_0}} = 1 \tag{4.68}$$

from which we get the following formula for adjusting the controller gain K_p:

$$K_p = K_{p_1} \frac{f_K(P_1)}{f_K(P)} \tag{4.69}$$

Adjusting K_p according to (4.69) ensures that the stability of the control loop is maintained for any P value.

Example 4.8 ***Model based adjustment of level controller***

Figure 4.27 shows a level control system for a cylindrical tank. You will

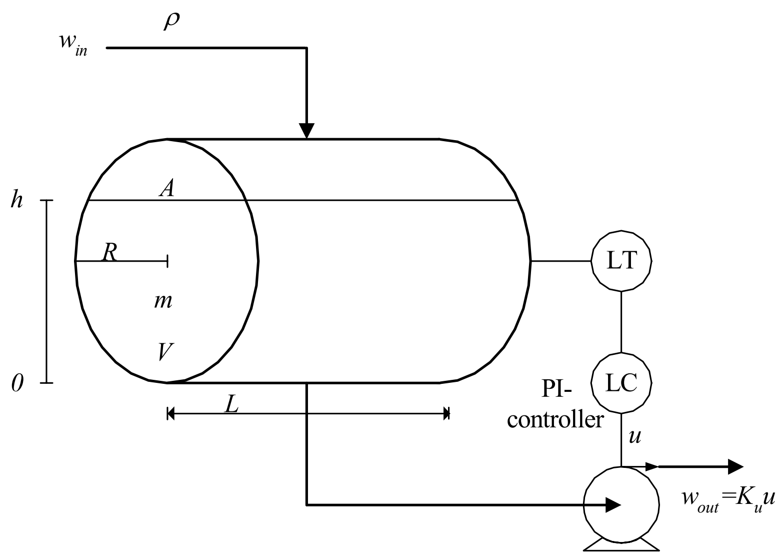

Figure 4.27: Example 4.8: Level control of a cylindric tank. The cross sectional area is a function of the level.

now see that the process gain K varies with the level. This implies that the controller gain K_p should vary. Mass balance for the liquid of the tank (we assume homogeneous conditions) is

$$\frac{dm}{dt} = \rho \frac{dV}{dt} = \rho A \frac{dh}{dt} = w_{in} - w_{out} = w_{in} - K_u u \tag{4.70}$$

It can be shown that the cross sectional area A is a function of the level h as follows:

$$A(h) = 2L\sqrt{R^2 - (R-h)^2} \tag{4.71}$$

From (4.70) we find that the transfer function from a deviation Δu in the control signal to the corresponding deviation Δh in the level is[11]

$$\frac{\Delta h(s)}{\Delta u(s)} = H(s) = -\frac{K_u}{\rho A(h)s} \tag{4.72}$$

giving the following process gain

$$K = -\frac{K_u}{\rho A(h)} = -\frac{K_u}{2\rho L\sqrt{R^2 - (R-h)^2}} = f_K(h) \tag{4.73}$$

The controller gain should be adjusted according to (4.69), which in this case gives

$$K_p = K_{p_1}\frac{f_K(h_1)}{f_K(h)} = K_{p_1}\frac{\left[-\frac{K_u}{\rho A(h_1)}\right]}{\left[-\frac{K_u}{\rho A(h)}\right]} = K_{p_1}\frac{A(h)}{A(h_1)} \tag{4.74}$$

where f_K is given by (4.73) and $A(h)$ is given by (4.71). K_{p_1} is a K_p value of a P or PI controller (the PID controller is not a good choice for this level control system since the process has pure integrator dynamics) tuned at some level h_1. (For example, h_1 may correspond to half of the maximum level.) K_p can be found by trial and error, or better: from transfer function based controller tuning, cf. Chapter 7. For example, (4.74) says that if the cross sectional area is halved (which gives doubled process gain), K_p should be halved. The integral time T_i in a PI controller can be unchanged in this case.

[End of Example 4.8]

4.9.4 Adaptive controller

In an adaptive control system, see Figure 4.28, a mathematical model of the process to be controlled is continuously estimated from samples of the control signal (u) and the process measurement (y_m). The model is typically a transfer function model. Typically, the structure of the model is fixed. The model parameters are estimated continuously using e.g. the least squares method. From the estimated process model the parameters of a PID controller (or of some other control function) are continuously

[11] The transfer function is here actually relating the deviation variables about an operating point since the process model is nonlinear.

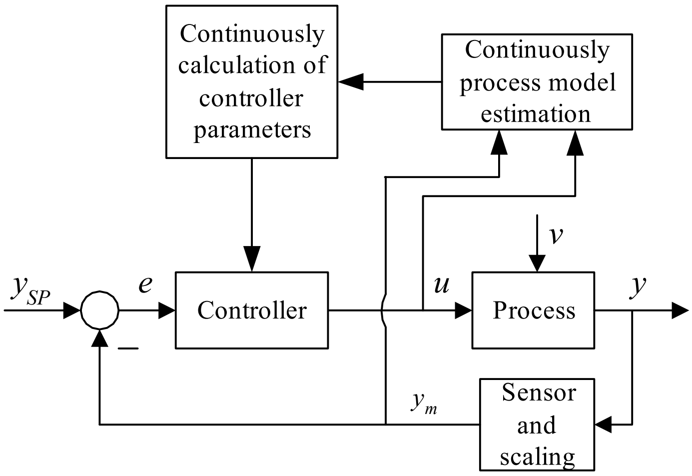

Figure 4.28: Adaptive control system

calculated so that the control system achieves specified performance in form of for example stability margins, poles, bandwidth, or minimum variance of the process output variable[22]. Adaptive controllers are commercially available, for example the ECA60 controller (ABB).