Figure 7.13 shows the simulated responses for the two control systems due to a setpoint step and a disturbance step. The dead-time compensator gives better setpoint tracking and better disturbance compensation than ordinary feedback control does.

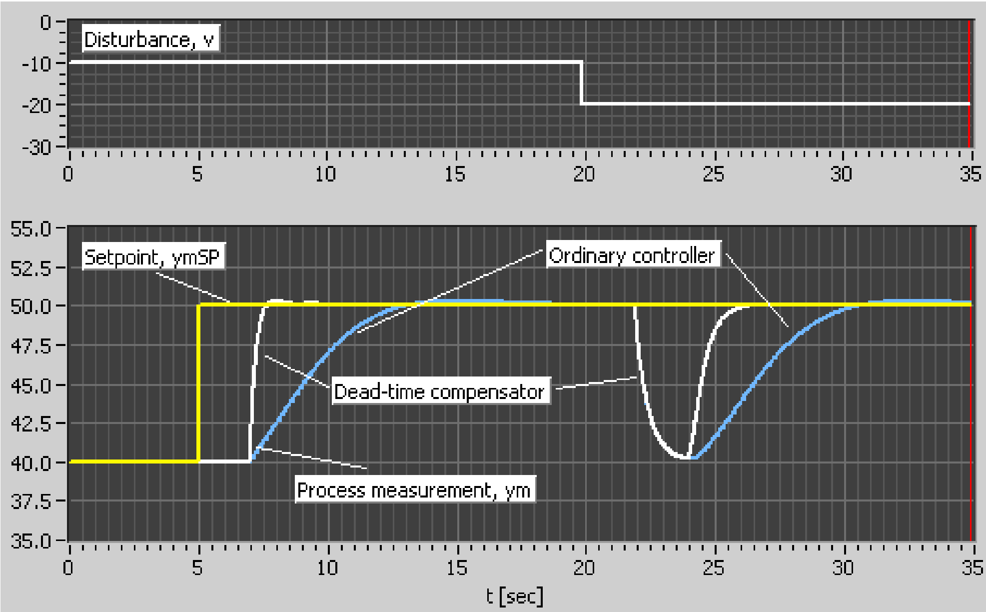

Figure 7.13: Example 7.6: Simulated responses for the two control systems due to a setpoint step and a disturbance step

[End of Example 7.6]

The dead-time compensator is model-based since the controller includes a model of the process. Consequently, the stability and performance robustness of the control system depend on the accuracy of the model. Running a sequence of simulations with a varied process model (changed model parameters) in each run is one way to investigate the robustness.

7.5 Skogestad's method

7.5.1 Introduction

[17] describes controller tuning for several types of transfer function processes – with and without time delay (dead-time). It is assumed that

the block diagram of the control system is as shown in Figure 7.2. The method, which can be denoted Skogestad's method after the originator[8], is based on the direct method described in Section 7.3: The control system tracking function $T(s)$ is specified as a first order transfer function with time delay:

$$T(s) = \frac{y_m(s)}{y_{m_{SP}}(s)} = \frac{1}{T_C s + 1} e^{-\tau s} \tag{7.87}$$

where T_C is the time constant of the control system which the user must specify, and τ is the process time delay which is given by the process model (the method can however be used for processes without time delay, too). Figure 7.14 shows the step response for (7.87).

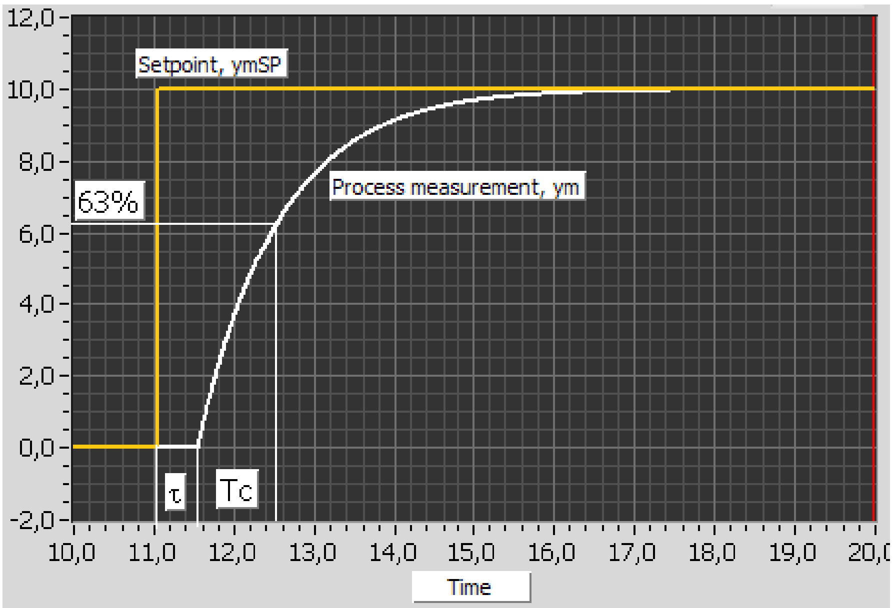

Figure 7.14: Step response of the specified tracking transfer function (7.87) in Skogestad's PID tuning method

The method is based on initially calculating the controller transfer function, $H_c(s)$, by (7.68) which is repeated here:

$$H_c(s) = \frac{1}{H_p(s)} \cdot \frac{T(s)}{1 - T(s)} \tag{7.88}$$

The process transfer function $H_p(s)$ may be of higher order than $T(s)$. Therefore, the specification (7.87) implies pole-zero cancellations in the control system loop transfer function, $L(s) = H_c(s)H_p(s)$. It is assumed

[8] Prof. Sigurd Skogestad

that the process $H_p(s)$ contains a time delay, $e^{-\tau s}$. The controller $H_c(s)$ according to (7.88) will contain the term $e^{-\tau s}$. This term is in $H_c(s)$ approximated by a first order Taylor series expansion which is $1 - \tau s$, and it turns out that the controller is a PI controller or a PID controller (depending on the process to be controlled).

Skogestad's method is in principle the same as dead-time compensation, which is described in Section 7.4, but in the latter there is no approximation of the time delay term. As with dead-time compensation Skogestad's method gives good setpoint tracking. The method gives formulas for the integral time, T_i, which are supposed to avoid slow disturbance compensation. In other controller design methods based on pole-zero cancellations there is a danger of slow disturbance compensation if the cancelled pole is close to zero (corresponding to cancellation of a large process time constant using a large T_i). This problem was demonstrated in Section 7.2.3.

The PID controller is assumed to be on serial form:

$$H_c(s) = K_p \frac{(T_i s + 1)(T_d s + 1)}{T_i s (T_f s + 1)} \tag{7.89}$$

If the PID controller you are going to apply is actually on parallel form,

$$H_c(s) = K_p + \frac{K_p}{T_i s} + \frac{K_p T_d s}{T_f s + 1} \tag{7.90}$$

you should consider *transforming* the PID parameters from serial form to parallel form to be sure that your parallel controller behaves like a serial controller. The transformation formulas are (2.51) – (2.53). (If the controller is a P or a PI controller, the transformation formulas need not be applied since in that case the serial and the parallel form are identical.)

7.5.2 Skogestad's tuning formulas

Skogestad's tuning formulas for several processes are shown in Table 7.1.[9] According to [17] the factor k_1 in Table 7.1 is 4, but there may be reasons to give it a different value, as argued on page 216. For the second order the process in Table 7.1 T_1 is the largest and T_2 is the smallest time constant.[10]

[9] [17] describes controller tuning for one additional process, namely a pure time delay, and the resulting controller is an I controller (Integral controller). However, a pure time delay can be approximated by a first order system with a small time constant (compared to the time delay), and this process is one of the processes in Table 7.1.

[10] [17] also describes methods for model reduction so that more complicated models can be approximated with one of the models shown in Table 7.1.

$H_p(s)$ (process)	K_p	T_i	T_d
$\frac{K}{s}e^{-\tau s}$	$\frac{1}{K(T_C+\tau)}$	$k_1\,(T_C+\tau)$	0
$\frac{K}{Ts+1}e^{-\tau s}$	$\frac{T}{K(T_C+\tau)}$	$\min\left[T,\, k_1\,(T_C+\tau)\right]$	0
$\frac{K}{(Ts+1)s}e^{-\tau s}$	$\frac{1}{K(T_C+\tau)}$	$k_1\,(T_C+\tau)$	T
$\frac{K}{(T_1s+1)(T_2s+1)}e^{-\tau s}$	$\frac{T_1}{K(T_C+\tau)}$	$\min\left[T_1,\, k_1\,(T_C+\tau)\right]$	T_2
$\frac{K}{s^2}e^{-\tau s}$	$\frac{1}{4K(T_C+\tau)^2}$	$4\,(T_C+\tau)$	$4\,(T_C+\tau)$

Table 7.1: Skogestad's formulas for PI(D) tuning. Standard value of k_1 is 4, but a smaller value, e.g. $k_1 = 1.44$ can give faster disturbance compensation. For the second order the process T_1 is the largest and and T_2 is the smallest time constant. (min means the minimum value.)

Unless you have reasons for a different specification, [17] suggests

$$T_C = \tau \tag{7.91}$$

to be used for T_C in Table 7.1.

The Ziegler-Nichols' closed loop method may be applied to most of the processes in Table 7.1 (since the processes have time delay). Generally, Skogestad's method results in better tracking property of the control system (without the quite large overshoot in the response after a step in the setpoint which is typical with Ziegler-Nichols' method), but the disturbance compensation may for some processes become more sluggish than with the Ziegler-Nichols' method. This sluggish compensation can however be speeded up by selecting a smaller value of k_1, cf. the discussion on page 216. It is here assumed that the disturbance is an input disturbance as explained on page 190.

Example 7.7 ***Control of first order system with time delay***

Let us try Skogestad's method and Ziegler-Nichols' closed loop method for tuning a PI controller for the process

$$H_p(s) = \frac{K}{Ts+1}e^{-\tau s} \tag{7.92}$$

where

$$K = 1;\, T = 0.5;\, \tau = 1 \tag{7.93}$$

(The time delay is relatively large compared to the time constant.) The controller parameters are as follows:

- Skogestad's method, cf. Table 7.1 with (7.91) and $k = 4$:

$$K_p = 0.25;\ T_i = 0.5 \tag{7.94}$$

- Ziegler-Nichols' closed loop method:

$$K_p = 0.68;\ T_i = 2.43 \tag{7.95}$$

Figure 7.15 shows control system responses for the two controller tunings. Skogestad's method works clearly better than Ziegler-Nichols' method, both with respect to setpoint tracking and disturbance compensation.

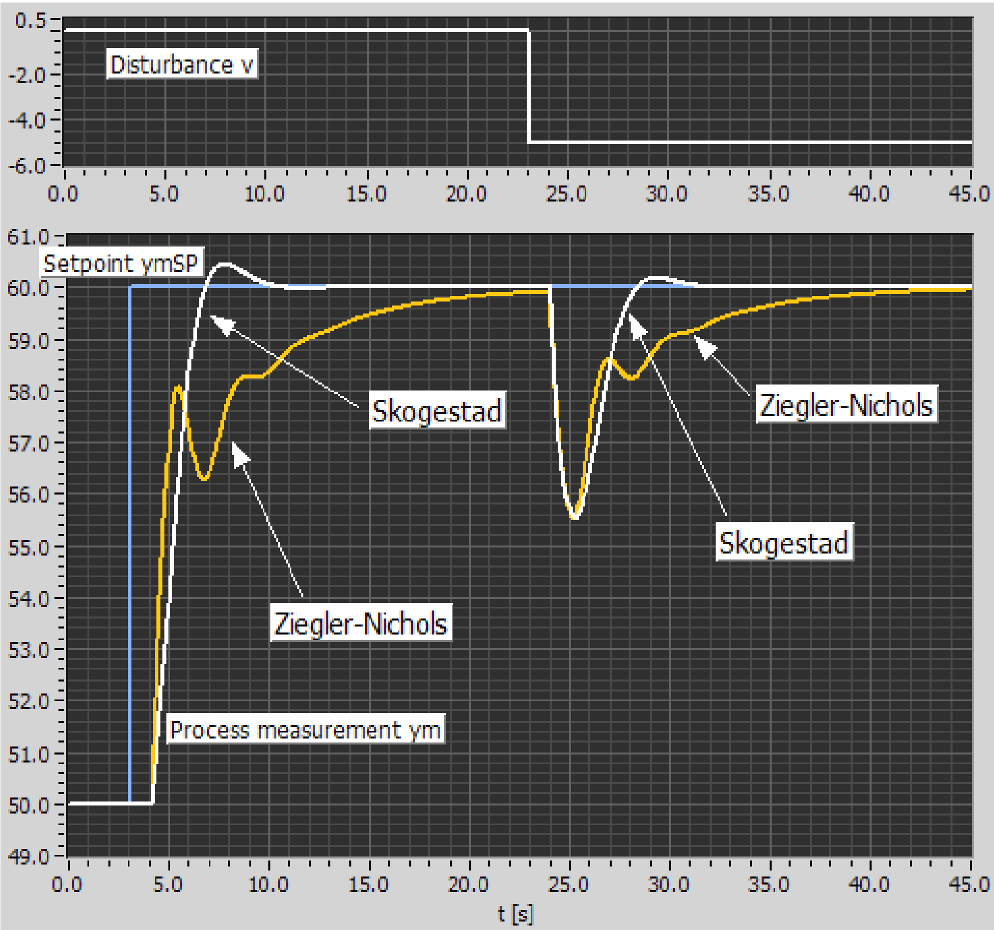

Figure 7.15: Example 7.7: Simulated responses in the control system for two different controller tunings

[End of Example 7.7]

7.5.3 Skogestad's method with faster disturbance compensation

According to [17], k_1 is 4 in Table 7.1. However, through simulations I have observed that $k_1 = 4$ in several cases gives quite sluggish disturbance compensation, although the parameter formulas in Table 7.1 are developed to avoid unnecessary sluggish compensation. A reduced k_1 value, as $k_1 = 1.44$, can give considerably faster disturbance compensation (since the integral time T_i is reduced).[11] A drawback of this modification of Skogestad's method is that there will be somewhat larger overshoot in the response after setpoint step, but in most cases such an increased overshoot is acceptable (if the setpoint is constant, which is typical, there is no overshoot, of course). Another drawback of the modification is that the stability robustness of the loop is somewhat reduced because of the reduced T_i.

Example 7.8 ***PI control of integrator with time delay***

The process

$$H_p(s) = \frac{K}{s} e^{-\tau s} \tag{7.96}$$

where

$$K = 1;\ \tau = 0.5 \tag{7.97}$$

will be controlled by a PI controller. (The wood-chip tank described in Example 2.3 has such a transfer function model.) Below are the PI parameters according to various tuning methods:

- Skogestad's method, cf. Table 7.1, with (7.91) and $k_1 = 4$:

$$K_p = 1;\ T_i = 4 \tag{7.98}$$

- Skogestad's method, cf. Table 7.1, with (7.91) and $k_1 = 1.44$:

$$K_p = 1;\ T_i = 1.44 \tag{7.99}$$

[11] According to [17] the standard value $k_1 = 4$ gives a transfer function from disturbance v to process measurement y_m in the control system with characteristic polynomial as of a critically damped second order system, i.e. the relative damping factor is $\zeta = 1$. This is quite a conservative choice. Faster but less damped dynamics is obtained with $\zeta < 1$. Simulations shows that $\zeta = 0.6$ is a reasonable value. It gives almost 3 times smaller T_i and therefore faster disturbance compensation. $\zeta = 0.6$ is obtained with $k_1 = 1.44$. It can be shown that the phase margin, PM, of a loop having second order characteristic polynomial is approximately equal to $100° \cdot \zeta$. With $\zeta = 0.6$ this equals $60°$ – a reasonable value in most cases.

- Ziegler-Nichols' closed loop method:

$$K_p = 1.3;\ T_i = 1.78 \tag{7.100}$$

Figure 7.16 shows simulated responses in the control system for the three different sets of PI parameter values. Skogestad's method with $k_1 = 4$ seems to give the best set point tracking, but there are no oscillations, indicating good (too good?) stability. The disturbance compensation with Skogestad's method with $k_1 = 4$ is clearly the slowest of the three alternatives.

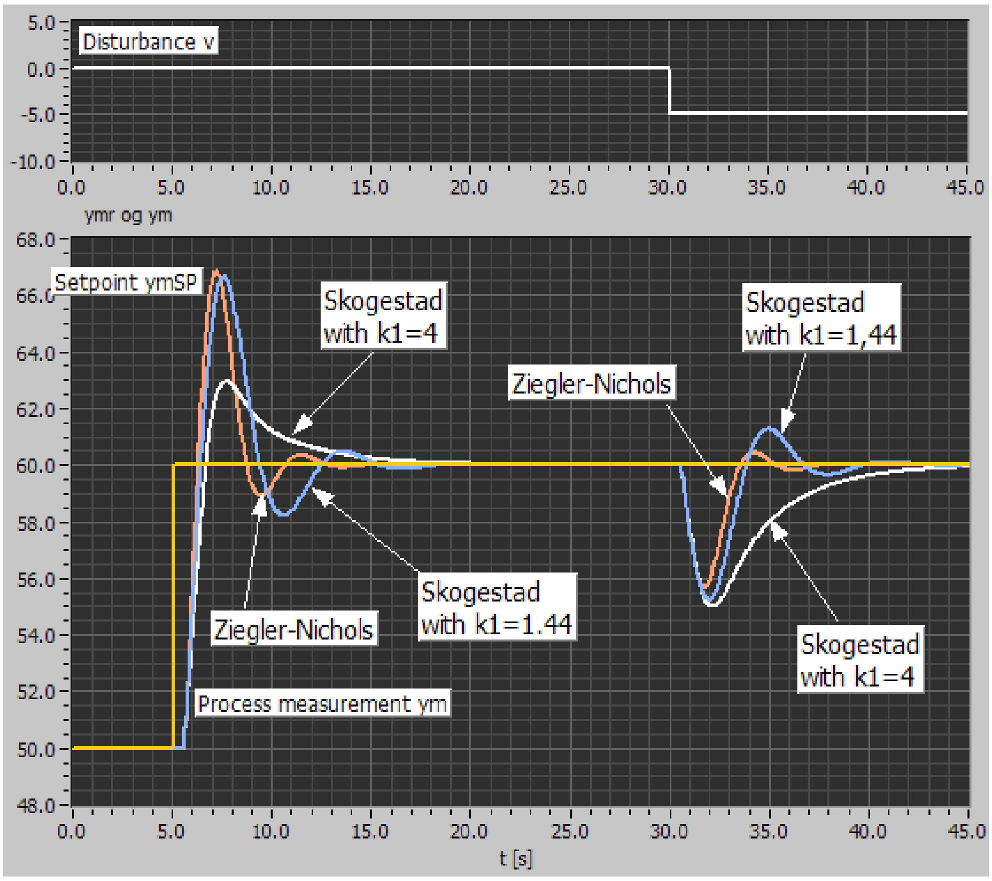

Figure 7.16: Example 7.8: Simulated responses in the control system for various PI tunings

[End of Example 7.8]

Example 7.7 demonstrated that it may be beneficial to set $k_1 = 1.44$ in stead of the standard value $k_1 = 4$ because faster disturbance is then

obtained. Let us review Example 7.7 which demonstrated that $k_1 = 4$ gave fast and properly damped disturbance compensation. Since $k_1 = 4$ worked well in that example, will the disturbance compensation in that example be worse with $k_1 = 1.44$ than with $k_1 = 4$? The answer is no, because: K_p is in any case independent of k_1, so it has value 0.25. However, T_i is dependent of k_1. According to Table 7.1,
$T_i = \min\left[T,\, k_1\left(T_C + \tau\right)\right] = \min\left[T,\, 2k_1\tau\right]$, but this minimum value is 0.5 no matter if k_1 is 4 or 1.44. So, this example has indicated that even if $k_1 = 4$ works fine, the suggestion $k_1 = 1.44$ makes no harm in this case.

7.5.4 Skogestad's method for processes without time delay

Each of the processes in Table 7.1 has time delay ($\tau > 0$). Can Skogestad's method be applied to processes *without* time delay? Yes, but in such cases we can not specify T_C according to (7.91) since τ is zero. We must specify T_C larger than zero. The controller parameter formulas are as shown in Table 7.2 (which is equal to Table 7.1 with $\tau = 0$).

$H_p(s)$ (process)	K_p	T_i	T_d
$\frac{K}{s}$	$\frac{1}{KT_C}$	$k_1 T_C$	0
$\frac{K}{Ts+1}$	$\frac{T}{KT_C}$	$\min\left[T,\, k_1 T_C\right]$	0
$\frac{K}{(Ts+1)s}$	$\frac{1}{KT_C}$	$k_1 T_C$	T
$\frac{K}{(T_1 s+1)(T_2 s+1)}$	$\frac{T_1}{KT_C}$	$\min\left[T_1,\, k_1 T_C\right]$	T_2
$\frac{K}{s^2}$	$\frac{1}{4K(T_C)^2}$	$4T_C$	$4T_C$

Table 7.2: Skogestad's formulas for PID tuning for processes without time delay. Standard value of k_1 is 4, but a smaller value, e.g. $k_1 = 1.44$ can give faster disturbance compensation. For the second order the process T_1 is the largest and and T_2 is the smallest time constant. (min means the minimum value.)

Example 7.9 ***PI control of first order system without time delay***

Given the following process:

$$H_p(s) = \frac{K}{Ts + 1} \tag{7.101}$$

where

$$K = 1;\, T = 5 \tag{7.102}$$

Let us specify $T_C = 1$. We try both $k_1 = 4$ (the standard value) and $k_1 = 1.44$ (which may give faster disturbance compensation). According to Table 7.2 the controller parameters (of a PI controller) are as follows:

- Skogestad's method, cf. Table 7.2, with $T_C = 1$ and $k_1 = 4$:

$$K_p = 5;\ T_i = 4 \tag{7.103}$$

- Skogestad's method, cf. Table 7.2, with $T_C = 1$ and $k_1 = 1.44$:

$$K_p = 1;\ T_i = 1.44 \tag{7.104}$$

Figure 7.17 shows simulated responses in the control system with the PI parameters values given above. We see that $k_1 = 1.44$ gives somewhat faster setpoint tracking, but with some overshoot, and in addition better disturbance compensation than with $k_1 = 4$.

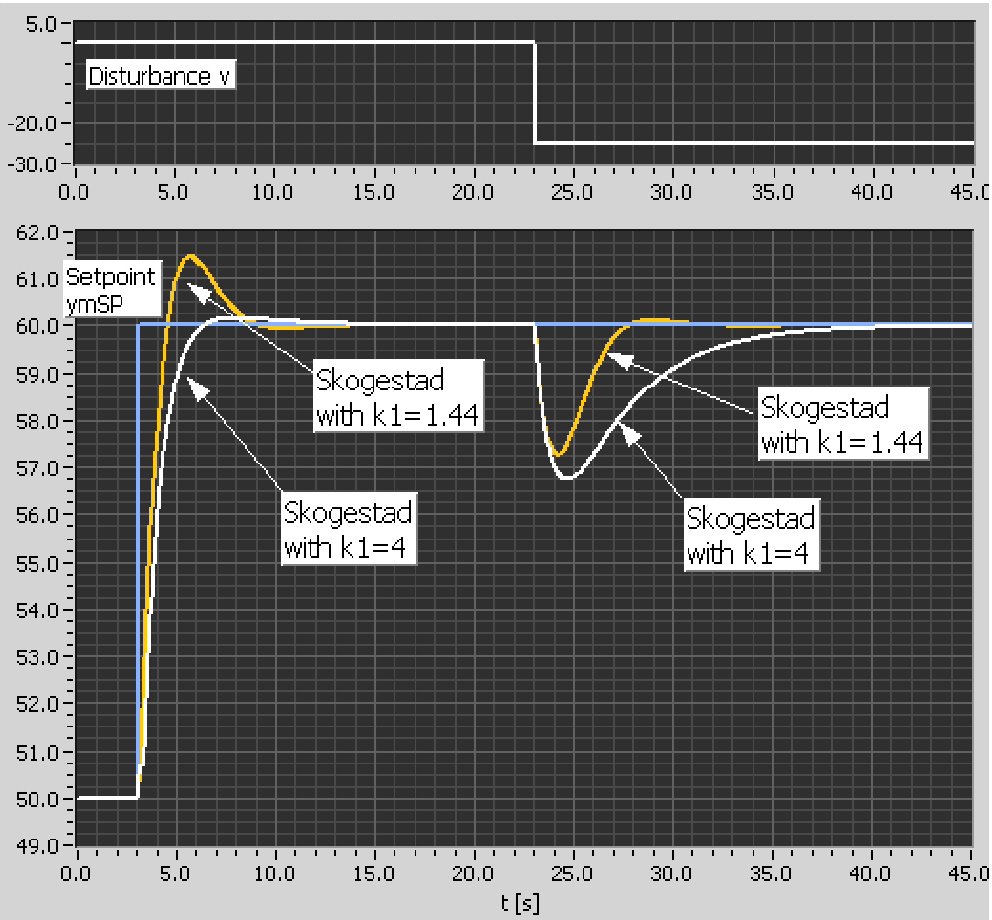

Figure 7.17: Example 7.9: Simulated responses in the control system for two different PI tunings

[End of Example 7.9]