Telemark University College/Finn Haugen

Solution to exam in Course PEF3006 Process Control

Exam date: 11. December 2014. Duration: 4 hours. Exam aids: None. Weight: 100% of course grade.

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1. The model is given as

$$c\rho V\dot{\theta}(t) = K_h u(t-\tau) + c\rho F\left[\theta_{in}(t) - \theta(t)\right] + U\left[\theta_e(t) - \theta(t)\right] \tag{1}$$

   (a) (10% weight) Regarding θ, θ_{in}, and θ_e as variables, Laplace transformation of Eq. (1) gives

$$c\rho V\left[s\theta(s) - \theta_0\right] = K_h u(s)e^{-\tau s} + c\rho F\left[\theta_{in}(s) - \theta(s)\right] + U\left[\theta_e(s) - \theta(s)\right] \tag{2}$$

   where θ_0 is the initial value of θ. From Eq. (2),

$$\begin{aligned}
\theta(s) &= \frac{c\rho V}{c\rho V s + c\rho F + U}\theta_0 && (3)\\
&+ \frac{K_h}{c\rho V s + c\rho F + U}e^{-\tau s}u(s) && (4)\\
&+ \frac{c\rho F}{c\rho V s + c\rho F + U}\theta_{in}(s) && (5)\\
&+ \frac{U}{c\rho V s + c\rho F + U}\theta_e(s) && (6)
\end{aligned}$$

   From expression (4), the transfer function from u to θ is

$$\underline{\underline{H(s) = \frac{\theta(s)}{u(s)} = \frac{K_h}{c\rho V s + c\rho F + U}e^{-\tau s}}} \tag{7}$$

   (b) (10%)

$$\begin{aligned}
H(s) &= \frac{K_h}{c\rho V s + c\rho F + U}e^{-\tau s} && (8)\\
&= \frac{\frac{K_h}{c\rho F + U}}{\frac{c\rho V}{c\rho F + U}s + 1}e^{-\tau s} && (9)\\
&= \frac{K}{Ts + 1}e^{-\tau s} && (10)
\end{aligned}$$

   Thus, the gain is

$$\underline{\underline{K = \frac{K_h}{c\rho F + U}}} \tag{11}$$

and the time constant is

$$T = \frac{c\rho V}{c\rho F + U} \tag{12}$$

2. (a) (5%) Figure 1 shows a P&I D of the control system.

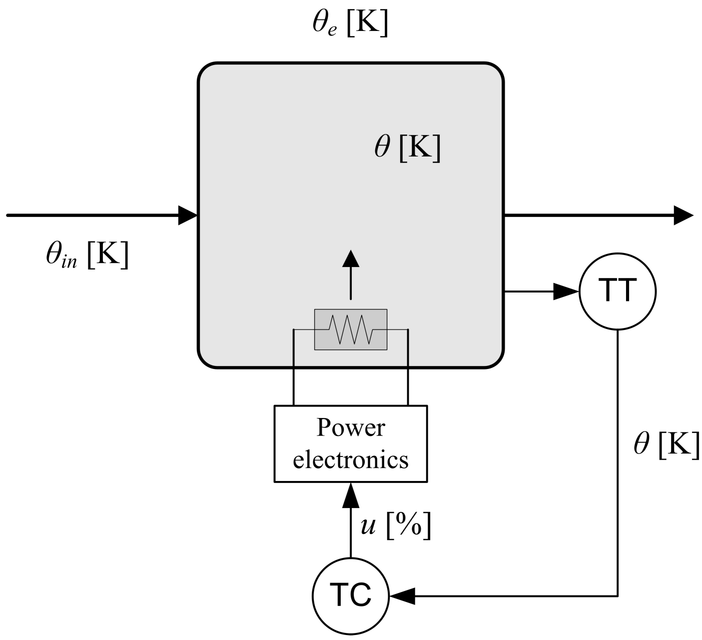

Figure 1: P&I diagram

(b) (10%) With $F = 0$ and $U = 0$, the process transfer function becomes

$$H(s) = \frac{K_h}{c\rho V s} e^{-\tau s} \tag{13}$$

$$= \frac{K_i}{s} e^{-\tau s} \tag{14}$$

where K_i is the integrator gain:

$$K_i = \frac{K_h}{c\rho V} \tag{15}$$

$H(s)$ is an "integrator with time-delay" process transfer function, and the Skogstad PI settings for such processes are given in the appendix of the exam text. Using Skogestad's hand-rule of selecting the close loop time-constant, namely $T_C = \tau$, the PI settings become

$$K_p = \frac{1}{2K_i \tau} = \frac{c\rho V}{2K_h \tau} \tag{16}$$

$$\underline{\underline{T_i = 4\tau}} \tag{17}$$

(c) (10%) The feedforward controller can be derived by substituting θ by its setpoint, θ_{SP}, in the model, Eq. (1), and then solving for the control signal, u. According to the problem formulation, $\tau = 0$. The result is the following formula of the control signal, i.e. the feedforward controller:

$$\underline{\underline{u_f(t) = \left\{ c\rho V \dot{\theta}_{SP}(t) - c\rho F \left[\theta_{in}(t) - \theta_{SP}(t)\right] - U \left[\theta_e(t) - \theta_{SP}(t)\right] \right\} / K_h}} \tag{18}$$

The following variables – assuming they are not known by other means – must be measured to make the feedforward controller implementable: F (with a flow sensor), θ_{in} (temperature sensor), and θ_e (temperature sensor). The other quantities, i.e. c, ρ, V, U, and K_h are here assumed known.

3. (10%) The Ziegler-Nichols method is used for the tuning. The ultimate gain is

$$K_{p_u} = 2 \tag{19}$$

and the ultimate period is

$$P_u = 60 \text{ s} \tag{20}$$

The PI settings become[1]

$$\underline{\underline{K_p = 0.45 K_{p_u} = 0.45 \cdot 2 = 0.9}} \tag{21}$$

$$\underline{\underline{T_i = \frac{P_u}{1.2} = \frac{60 \text{ s}}{1.2} = 50 \text{ s}}} \tag{22}$$

4. (5%) Most PID controllers are used as PI controllers in practical control systems because the derivative term propagates measurement noise, causing the control signal to become noisy, which may cause excessive wear of the actuator.

5. (15%) Figure 2 shows a general block diagram of a cascade control system. The control signal calculated by the primary controller is used as the setpoint of the secondary controller. The secondary control loop compansates quickly for the disturbance so that the disturbance does not influence the primary process output variable. The main benefit of cascade control comparing with single loop control is the ability to more quickly and effectively compensate for process distrurbances.

[1] Full score is given if only the intermediate answers, 0.9 and 50 s, respectively, are given (in the lack of a calculator or a computer :-)

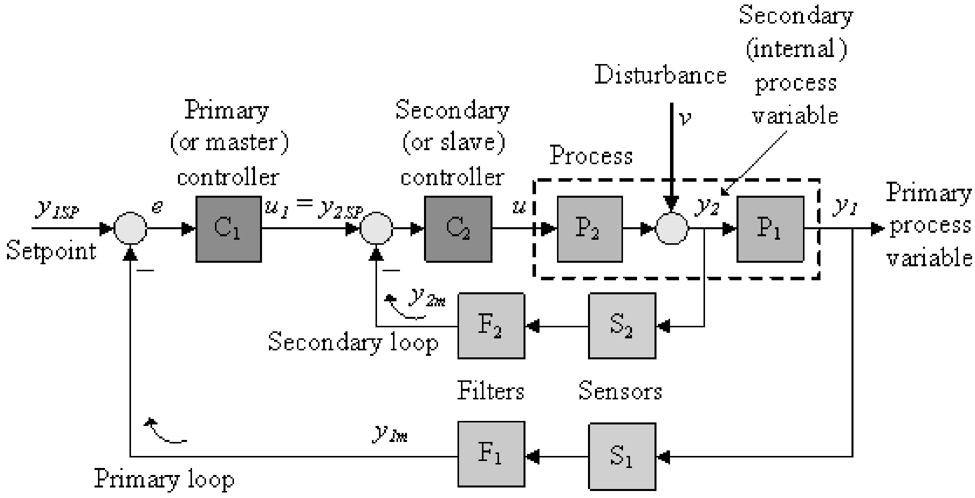

Figure 2: Cascade control system

6. (10%) Figure 3 shows a level control with a control valve assumed to provide increased opening, and hence increased flow, if the control signal to the valve is increased (hence, the valve is a Fail Closed valve). In this control system the controller must have direct action.

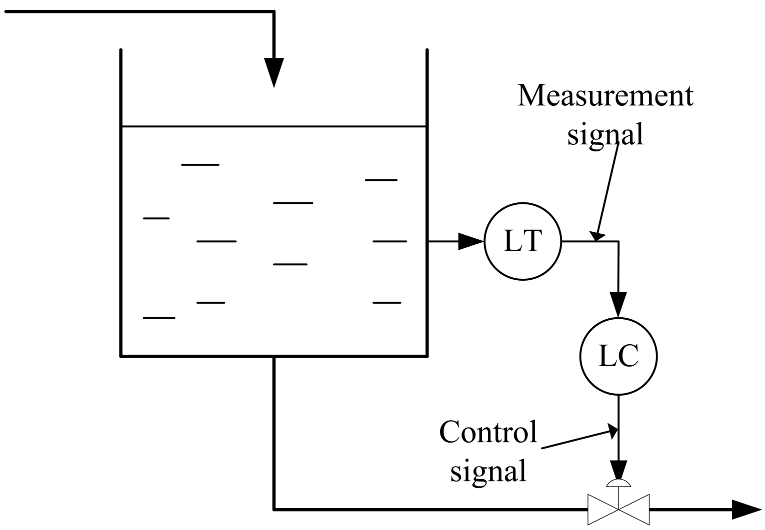

Figure 3: Tank with level control system

Explanation of why direct action is needed: Assume that the level (the process variable) is at the setpoint, and that the level then (for some reason) increases. To get the level back to the setpoint, the outflow must be increased. With the given valve, increased flow is obtained with increased control signal. In other words, in this system, a process variable (or measurement) *increase* requires a control signal *increase*, and therefore, the controller must have *direct action*.

The consequence of selecting wrong among direct and reverse action mode is that the control system becomes *unstable*.

7. (5%) In Figure 4, the left diagram shows the structure of a ratio control system in detail, while the right diagram shows a compact

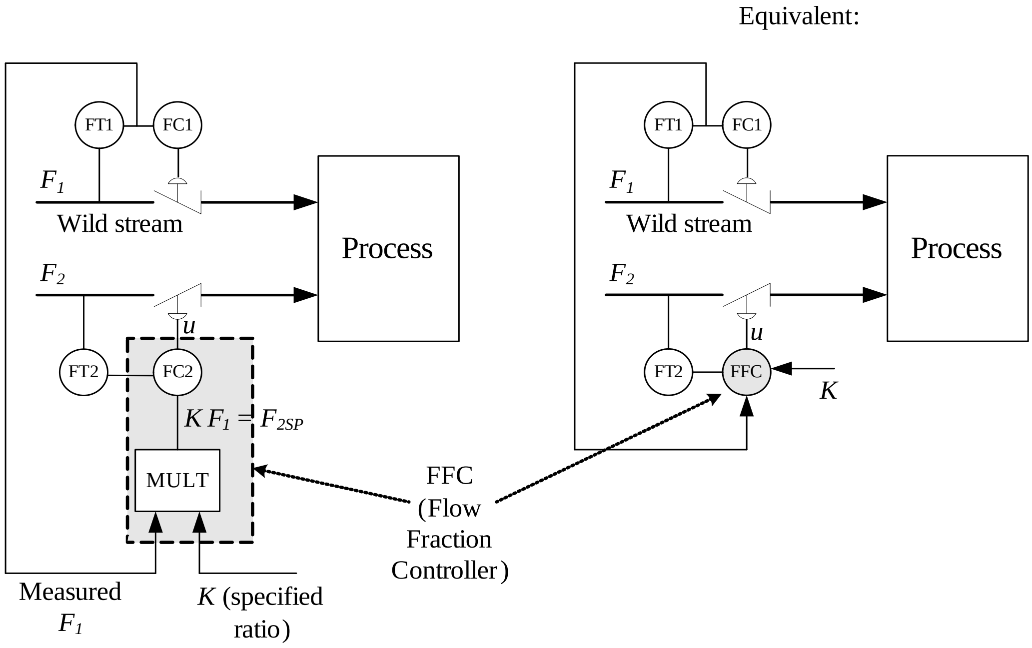

Figure 4: Ratio control

but equivalent representation of ratio control with the symbol FFC (Flow Fraction Controller).

The purpose of ratio control is to control a mass flow, F_2 in Figure 4, so that the ratio between this flow and another flow, F_1, is

$$F_2 = KF_1 \tag{23}$$

where K is a specified ratio.

8. (10%) The (three) main elements of a sequential function chart (SFC) are:

- Steps defines the possible states of the control system. A step is either active or passive.

- Actions of a step are the control actions executed by the control device (typically a PLC), e.g. opening a valve, when that step is active.
- Transitions are the jumps from presently active steps to their next steps. A transition from an active step to a next step takes place once the transition condition is satisfied, e.g. once a button has been pressed, or once the level in a tank has passed a certain value.