

Exercises to Basic Dynamics and Control

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TechTeach

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Preface

This book contains exercises with solutions to the text-book *Basic Dynamics and Control*, TechTeach, August 2010. The exercises can all be solved with just manual calculations (using paper and pencil). So, computer-based exercises are not covered.¹

The following freely available material may also be useful in exercises. This material is available at <http://techteach.no>.

- **SimView** which is a collection of ready-to-run simulators.
- **TechVids** which is a collection of instructional streaming videos, together with the simulators that are played and explained by me as a part of the videos.

This book is organized in chapters and sections which corresponds to the text-book.

Finn Haugen, MSc

TechTeach

Skien, Norway, August 2010

¹The formulation of computer-based exercises depends largely on the tool being used, e.g. MATLAB/SIMULINK, LabVIEW, Scilab/Scicos, Octave., and the tool being used may vary from one school/university to another. Therefore, this book does not contain such exercises.

Part I

EXERCISES

Chapter 1

Introduction to control

Exercise 1.1

Figure 1.1 shows the different components of a speed control system of an electric motor.

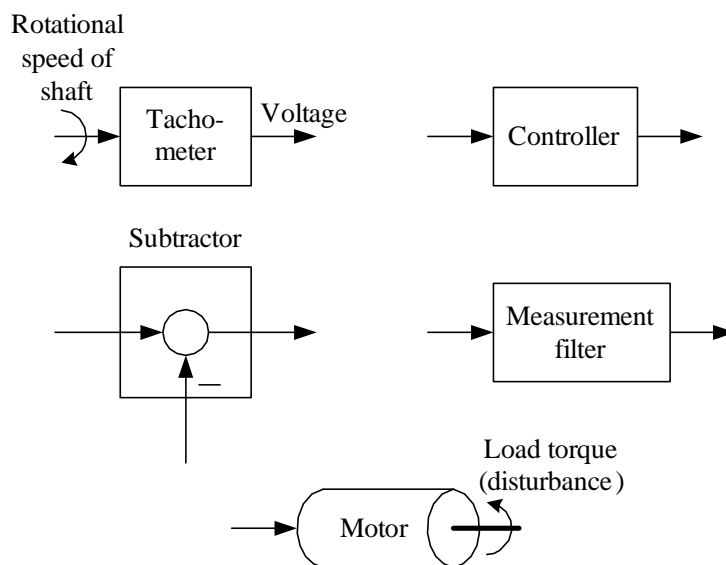


Figure 1.1:

1. “Construct” a speed control system by connecting the components (draw a block diagram of the control system). Where is the control

error in your block diagram?

2. How does the control system work? (Assume that the speed initially is equal to the speed reference (setpoint), and that the load torque is increased so that the motor speed is reduced.)

Exercise 1.2

The weight control system shown in Figure 1.2 seems to be an automatic bartender.¹ Explain how the control system works. (Explain the feedback

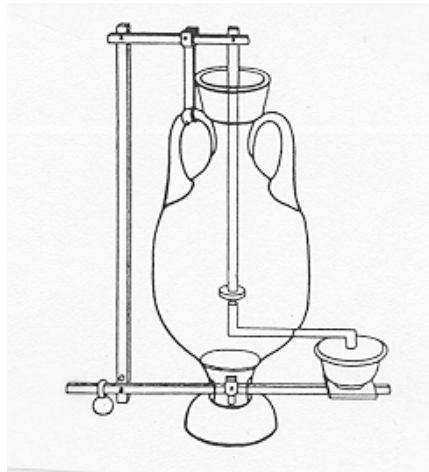


Figure 1.2: A weight control system from the Antics. An automatic bartender?
control action.)

Do you know any other process of your daily life which uses the same principle of level control?

Exercise 1.3

Figure 1.3 shows an evaporator where the product is created by evaporating the feed. (As an example, an evaporator is used to remove water from the half-finished fertilizer in a fertilizer factory.) Suggest a control structure for the evaporator according to the following requirements: The feed is a “wild stream”, i.e. it shall not be controlled (in this exercise). The mass balances of the liquid and the vapour must be maintained. The product quality or composition is strongly influenced by the steam supply (flow).

¹From the book “Origins of Feedback Control” by O. Mayr.

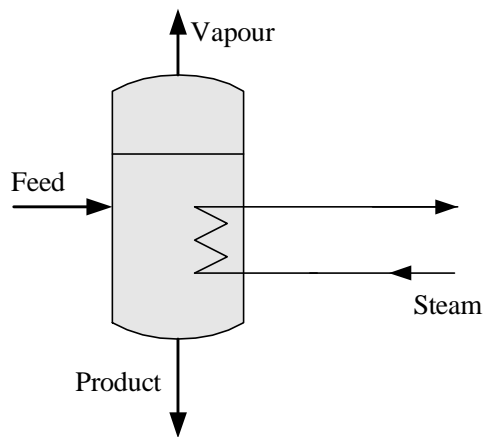


Figure 1.3:

Exercise 1.4

Below are mentioned several processes which are supposed to be controlled. The process output variable is indicated in paranthesis. For each of the processes, what is

- control (manipulating) variable
- disturbances or loads or environmental variables (these are alternative names)

Make your own assumptions. The processes are as follows:

1. Robot arm or manipulator driven by an electric motor (arm position)
2. Steam heated heat exchanger with some process fluid to be heated (temperature of fluid outlet)
3. Ship positioned with thrusters (denoted a dynamic positioning system) (ship position)

Exercise 1.5

Control engineering may be crucial for each of the following kinds of applications. For each of these, give one concrete example of a system,

including identification of the variable to be kept under control. (For example, the autopilot of an aircraft may be crucial for safety. Aircraft position and speed are controlled.)

- Product quality
- Product economy
- Safety
- Environmental care
- Comfort
- Feasibility
- Automation

Chapter 2

Representation of differential equations with block diagrams and state-space models

2.1 Introduction

No exercises here.

2.2 Dynamic systems

Exercise 2.1

Give three examples of dynamic systems, and for each system, identify at least one input variable and at least one output variable.

2.3 Mathematical block diagrams

Exercise 2.2

Figure 2.1 shows a liquid tank. Assume that the following differential

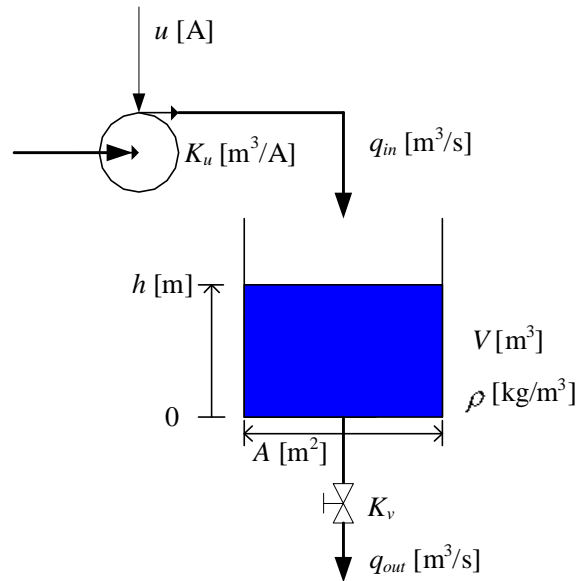


Figure 2.1:

equation is a mathematical model describing how the level h varies:

$$A\dot{h}(t) = K_u u(t) - K_v \sqrt{\rho g h(t)} \quad (2.1)$$

where A , K_u , K_v , ρ and g are parameters. The initial value of h is h_0 . Pump inflow is $K_u u$ where u is the pump control signal. Valve outflow is $K_v \sqrt{\rho g h(t)}$ where K_v is the valve constant. h is the level. ρ is density. A is cross-sectional area. g is gravity. h is the output variable of the system, and u is the input (manipulating) variable.

1. Draw a detailed mathematical block diagram of the differential equation (2.1) using blocks (no formula block).
2. Draw a block diagram using a formula block in stead of blocks as in Problem 1 (but still use an integrator block).
3. What use can you have of such a block diagram?

2.4 State-space models

Exercise 2.3

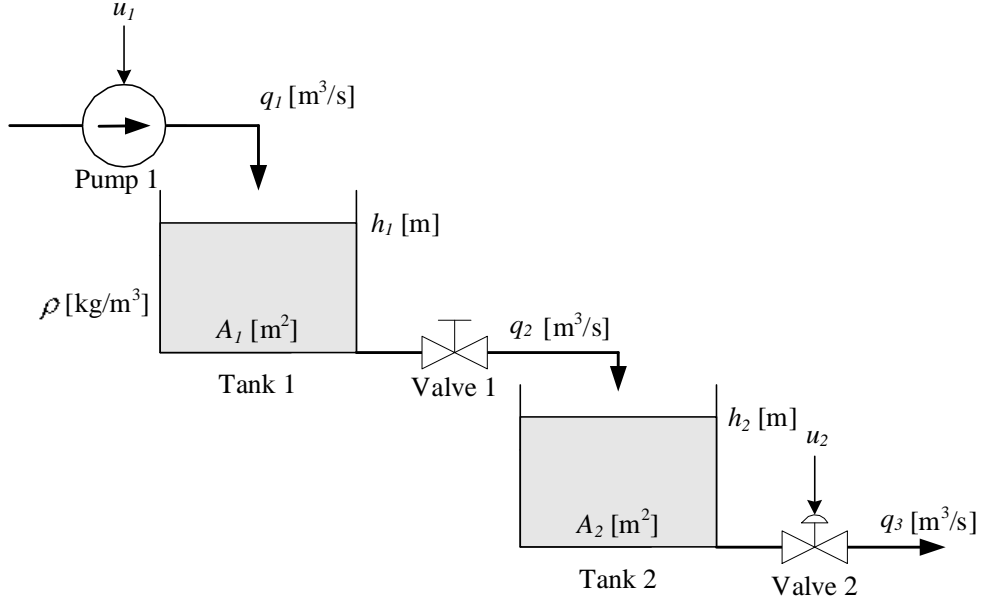


Figure 2.2:

Figure 2.2 shows two coupled liquid tanks. u_1 and u_2 are control signals. Mass balance of tank 1 is

$$\rho A_1 \dot{h}_1 = \underbrace{\rho K_p u_1}_{q_1} - \underbrace{\rho K_{v_1} \sqrt{\frac{\rho g h_1}{G}}}_{q_2} \quad (2.2)$$

Mass balance of tank 2 is

$$\rho A_2 \dot{h}_2 = \underbrace{\rho K_{v_1} \sqrt{\frac{\rho g h_1}{G}}}_{q_2} - \underbrace{\rho K_{v_2} u_2 \sqrt{\frac{\rho g h_2}{G}}}_{q_3} \quad (2.3)$$

Valve 1 has fixed opening. Valve 2 is a control valve with control signal u between 0 and 1. The square root functions stems from the common valve characteristic which expresses that the flow is proportional to the square root of the pressures drop across the valve. Here, the pressure drops are assumed to be equal to the hydrostatic pressures at the bottom the tanks. For example, for tank 1 the hydrostatic pressure is $\rho g h_1$. The parameter G is the relative density of the liquid.¹

Assume that the input variables are u_1 and u_2 , and that the output variables are $y_1 = h_1$ and $y_2 = h_2$. Write the model (2.2) – (2.3) as a state-space model.

¹ $G = \rho / \rho_{water}$.

2.5 How to calculate static responses

Exercise 2.4

See Exercise 2.2. Assume that all the variables in (2.1) have constant values, which you can indicate with sub-index “s” in your answer.

Find the constant pump control signal u_s that gives steady-state level h_s .

Chapter 3

Mathematical modeling

3.1 Introduction

No exercises here.

3.2 A procedure for mathematical modeling

Exercise 3.1

The inventory at time t is given by

$$\text{Inventory}(t) = \text{Inventory}(0) + \int_0^t (\text{Inflows} - \text{Outflows} + \text{Generated}) d\theta \quad (3.1)$$

Draw a mathematical block diagram of (3.1).

3.3 Mathematical modeling of material systems

Exercise 3.2

Figure 3.1 shows a wood chip tank with a feed screw and conveyor belt (the belt has constant speed).¹ There is an outflow of chip via an outlet at

¹Typically, there is such a chip tank in the beginning of the production line of a paper mass factory.

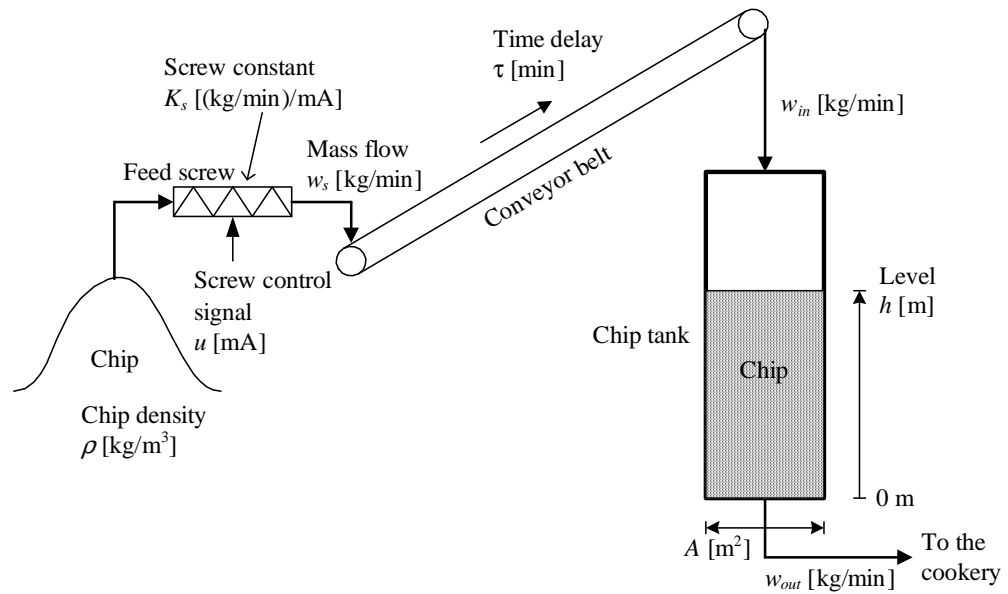


Figure 3.1:

the bottom of the tank. The mass flow w_s from the feed screw to the belt is proportional to the screw control signal u :

$$w_s = K_s u \quad (3.2)$$

The mass flow w_{in} into the chip tank is equal to w_s but time delayed time τ :

$$w_{in}(t) = w_s(t - \tau) \quad (3.3)$$

1. Draw an overall input-output block diagram of the system. Define the input and output variables (it is assumed that the level is of particular interest).
2. Develop a mathematical model describing the behaviour of the chip level.
3. Draw a detailed mathematical block diagram from the derived mathematical model. Assume that all independent variables and parameters are input variables.

3.4 Mathematical modeling of thermal systems

Exercise 3.3

Figure 3.2 shows a tank with cold water inflow and heated (blended) water outflow. The tank is full constantly, and the volumetric flow is thus equal to the sum of the inflows. Assume homogenous conditions in the tank.

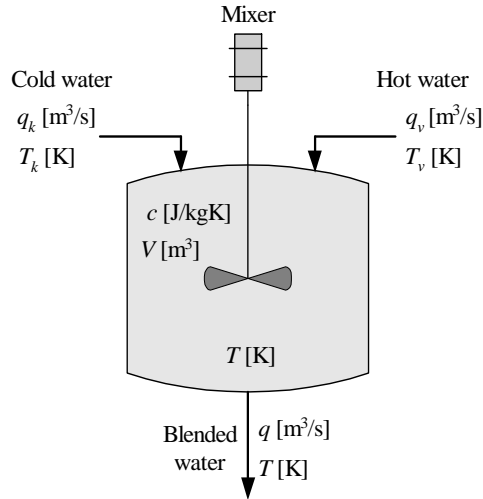


Figure 3.2:

Develop a mathematical model of the water temperature T in the tank.

3.5 Mathematical modeling of motion systems

Exercise 3.4

Figure 3.3 shows a ship. In this exercise we concentrate on the so-called surge (forward-backward) direction, i.e., the movements in the other directions are disregarded. The wind acts on the ship with the force F_w . The hydrodynamic damping force F_h (damping from the water) is proportional to the square of the difference between the ship speed u and the water current speed u_c . Assume that the proportionality constant is D (a positive number).

1. What is the mathematical relation between speed u and position x ?
2. Develop a mathematical model of the ship expressing the motion (the position) in the surge direction.
3. Draw an input-output block diagram of the system. Assume that the ship position is the variable of particular interest.

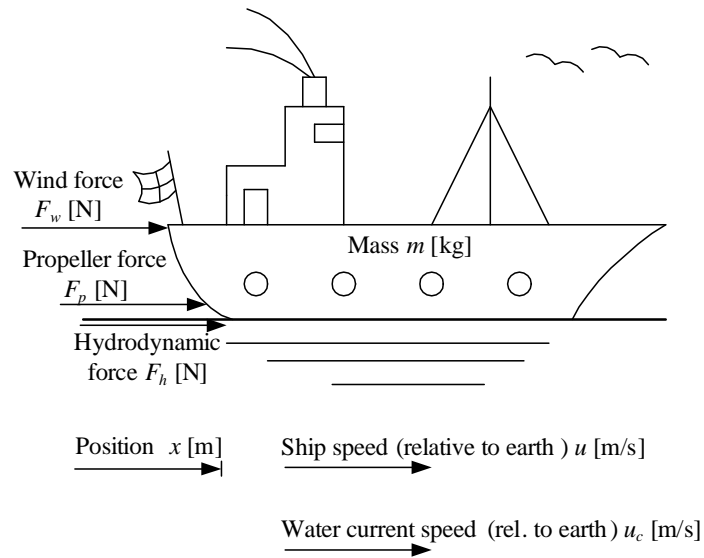


Figure 3.3:

Exercise 3.5

Figure 3.4 shows a satellite with manoeuvring motors. Develop the equation of the motion of the satellite.

Exercise 3.6

Figure 3.5 shows a cart with the pendulum. A motor (in the cart) acts on the cart with a force F .² You can use the following variables and parameters in the model:

- I – the moment of inertia of the pendulum about its center of gravity. For the pendulum shown in Figure 1,

$$I = \frac{mL^2}{12} \quad (3.4)$$

- V and H – vertical and horizontal forces, respectively, in the pivot.

- d – a damping coefficient.

²This force can be manipulated by the controller to stabilize the pendulum in an upright position or in a downright position at a specified position of the cart, but this exercise is not about control. The system can be well controlled with model-based control, for example optimal control based on state-variable feedback (cf. e.g. Lecture notes on Modes, Estimation and Control, TechTeach/F. Haugen).

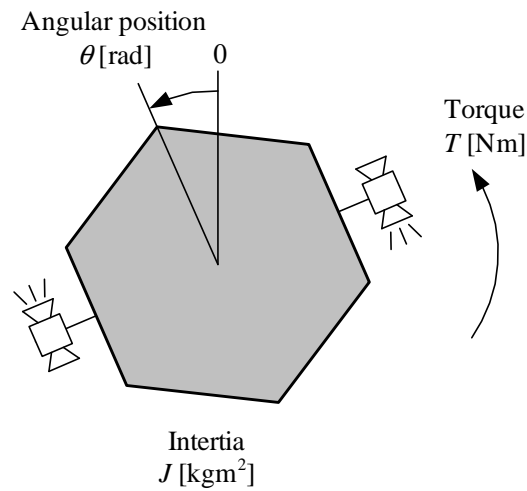


Figure 3.4:

Derive a mathematical model of the system based on the following principles:

1. Force balance (Newton's Second Law) applied to the horizontal movement of the center of gravity of the pendulum.
2. Force balance applied to the vertical movement of the center of gravity of the pendulum.
3. Torque balance (the rotational version of the Newton's Second Law applied to the center of gravity of the pendulum).
4. Force balance applied to the cart.

(When using the model for developing a simulator or design of a stabilizing controller, it will probably be necessary to eliminate the internal forces V and H , but this elimination is not a part of this exercise. Hence, it is ok that the resulting model in this exercise contains V and H .)

3.6 Mathematical modeling of electrical systems

Exercise 3.7

Figure 3.6 shows a combination of resistors. What is the resulting

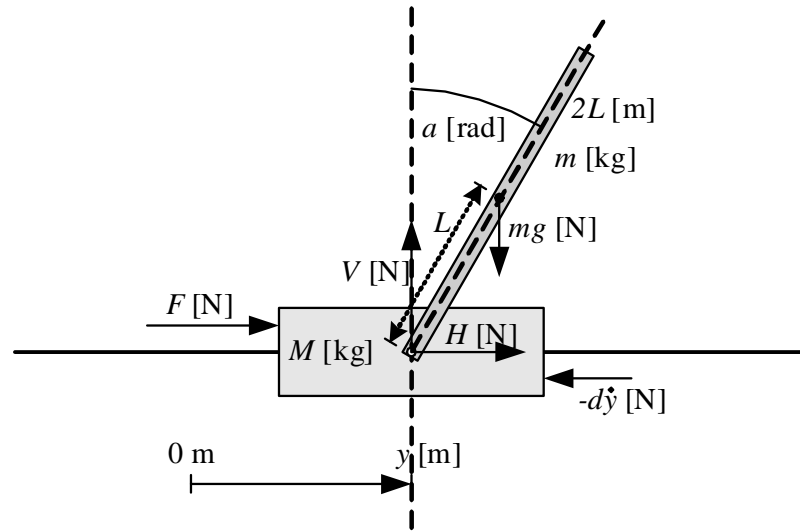


Figure 3.5:

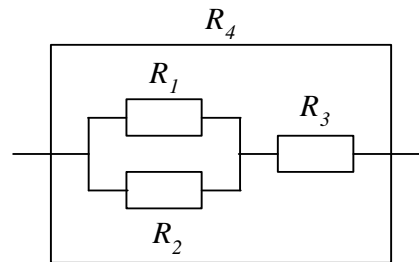


Figure 3.6:

resistance R_4 ?

Exercise 3.8

Given a lamp which receives $P = 100 \text{ W}$ mean (average) power when it is connected to the mains, which is an alternate voltage of amplitude $U = 220 \text{ V}$. Calculate the lamp resistance R .

Exercise 3.9

Figure 3.7 shows an (analog) highpass filter. (It attenuates low-frequency signals, while high-frequency signals pass through the filter.) Find a

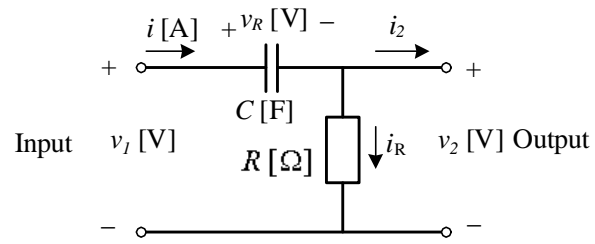


Figure 3.7:

mathematical model that relates the input voltage v_1 and the output voltage v_2 .

Chapter 4

The Laplace transform

Exercise 4.1

Calculate the Laplace transform, $F(s)$, of the time function

$$f(t) = e^{-t} \quad (4.1)$$

using the definition of the Laplace transform.

Can you find the same answer ($F(s)$) by using a proper Laplace transform pair?

Exercise 4.2

Given the following differential equation:

$$\dot{y}(t) = -2y(t) + u(t) \quad (4.2)$$

with initial value $y(0) = 4$. Assume that the input variable $u(t)$ is a step of amplitude 1 at time $t = 0$.

1. Calculate the response in the output variable, $y(t)$, using the Laplace transform.
2. Calculate the steady-state value of $y(t)$ using the Final Value Theorem. Also calculate the steady-state value, y_s , from $y(t)$, and from (4.2) directly. Are all these values of y_s the same?

Chapter 5

Transfer functions

5.1 Introduction

No exercises here.

5.2 Definition of the transfer function

Exercise 5.1

In Exercise 3.2 the mathematical model of a wood-chip tank was derived. The model is

$$\rho A \dot{h}(t) = K_s u(t - \tau) - w_{out}(t) \quad (5.1)$$

Calculate the transfer function $H_1(s)$ from the screw control signal u to the level h and the transfer function $H_2(s)$ from the outflow w_{out} to the level h . (Tip: Use Eq. (4.16) in the text-book to calculate the Laplace transform of the time-delay.)

Exercise 5.2

Figure 5.1 shows a mass-spring-damper-system. y is position. F is applied force. D is damping constant. K is spring constant. It is assumed that the damping force F_d is proportional to the velocity, and that the spring force F_s is proportional to the position of the mass. The spring force is assumed to be zero when y is zero. Force balance (Newton's 2. Law) yields

$$m\ddot{y}(t) = F(t) - D\dot{y}(t) - Ky(t) \quad (5.2)$$

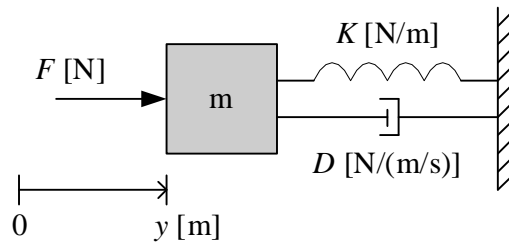


Figure 5.1:

Calculate the transfer function from force F to position y .

5.3 Characteristics of transfer functions

Exercise 5.3

Given the following transfer function:

$$H(s) = \frac{s + 3}{s^2 + 3s + 2} \quad (5.3)$$

1. What is the order?
2. What is the characteristic equation?
3. What is the characteristic polynomial?
4. What are the poles and the zeros?

5.4 Combining transfer functions blocks in block diagrams

Exercise 5.4

Given a thermal process with transfer function $H_p(s)$ from supplied power P to temperature T as follows:

$$T(s) = \underbrace{\frac{b_p}{s + a_p}}_{H_p(s)} P(s) \quad (5.4)$$

The transfer function from temperature T to temperature measurement T_m is as follows:

$$T_m(s) = \frac{b_m}{\underbrace{s + a_m}_{H_m(s)}} T(s) \quad (5.5)$$

a_p , b_p , a_m , and b_m are parameters.

1. Draw a transfer function block diagram of the system (process with sensor) with P as input variable and T_m as output variable.
2. What is the transfer function from P to T_m ? (Derive it from the block diagram.)

5.5 How to calculate responses from transfer function models

Exercise 5.5

Given the transfer function model

$$y(s) = \frac{5}{\underbrace{s}_{H(s)}} u(s) \quad (5.6)$$

Suppose that the input u is a step from 0 to 3 at $t = 0$. Calculate the response $y(t)$ due to this input.

5.6 Static transfer function and static response

Exercise 5.6

See Exercise 5.2. It can be shown that the transfer function from force F to position y is

$$H(s) = \frac{y(s)}{F(s)} = \frac{1}{ms^2 + Ds + K} \quad (5.7)$$

Calculate the static transfer function H_s . From H_s calculate the static response y_s corresponding to a constant force, F_s .

Chapter 6

Dynamic characteristics

6.1 Introduction

No exercises here.

6.2 Integrators

Exercise 6.1

See Exercise 5.1. The transfer function from w_{out} to h is

$$\frac{h(s)}{w_{out}(s)} = -\frac{1}{\rho A s} = H_2(s) \quad (6.1)$$

1. Does this transfer function represent integrator dynamics?
2. Assume that $w_{out}(t)$ is a step from 0 to W at time $t = 0$. Calculate the response $h(t)$ that this excitation causes in the level h . You are required to base your calculations on the Laplace transform.

Exercise 6.2

Figure 6.1 shows an isolated tank (having zero heat transfer through the walls). Show that the tank dynamically is an integrator with the power P as input variable and the temperature T as output variable. (Hint: Study the transfer function from P to T .)

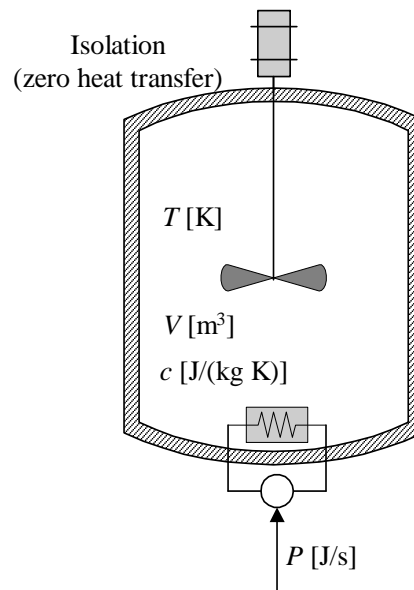


Figure 6.1:

6.3 Time-constants

Exercise 6.3

Calculate the gain and the time-constant of the transfer function

$$H(s) = \frac{y(s)}{u(s)} = \frac{2}{4s + 8} \quad (6.2)$$

and draw by hand roughly the step response of $y(t)$ due to a step of amplitude 6 in u from the following information:

- The steady-state value of the step response
- The time-constant
- The initial slope of the step response, which is

$$S_0 = \dot{y}(0^+) = \frac{KU}{T} \quad (6.3)$$

(This can be calculated from the differential equation (6.12) in the text-book by setting $y(0) = 0$.)

Exercise 6.4

Figure 6.2 shows the temperature response T of a thermal system due to a step of amplitude 1 kW in the supplied power P . Find the transfer

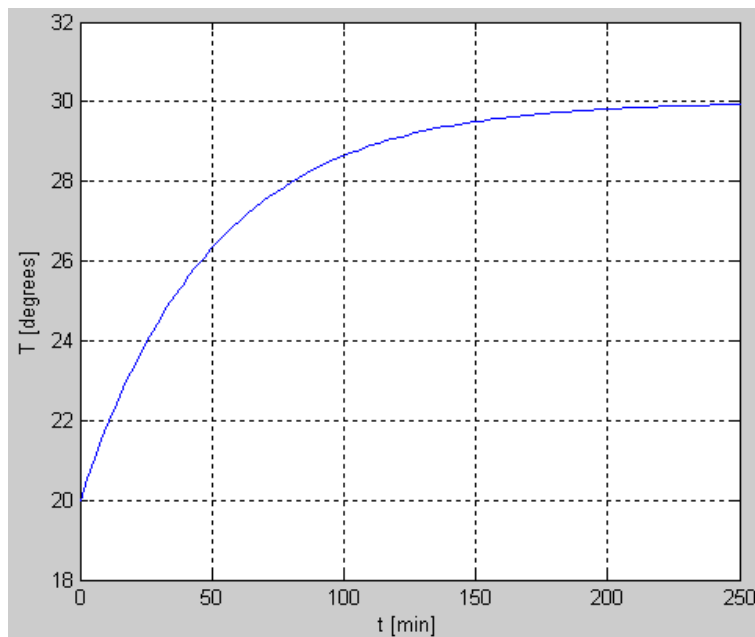


Figure 6.2:

function from ΔP (power) to ΔT (temperature) where Δ indicates deviations from the steady-state values. Assume that the system is of first order (a time-constant system).

Exercise 6.5

Figure 6.3 shows an RC-circuit (the circuit contains the resistor R and the capacitor C). The RC-circuit is frequently used as an analog lowpass filter: Signals of *low* frequencies *passes* approximately unchanged through the filter, while signals of high frequencies are approximately filtered out (stopped). It can be shown that a mathematical model of the RC circuit is

$$RC\dot{v}_{in} = v_{out} - v_{in} \quad (6.4)$$

1. Calculate the transfer function $H(s)$ from v_{in} to v_{out} , and calculate the gain and the time-constant of $H(s)$.

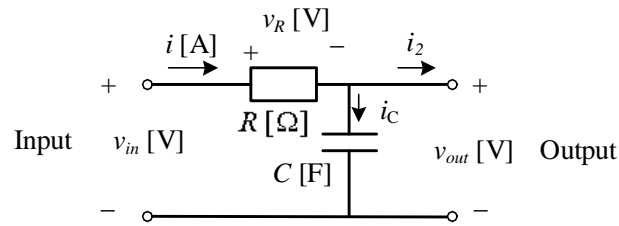


Figure 6.3: RC-circuit

2. Assume that the RC circuit is used as a signal filter. Assume that the capacitance C [F] is fixed. How can you adjust the resistance R (increase or decrease) so that the filter performs stronger filtering or, in other words: is more sluggish.

6.4 Time-delays

Exercise 6.6

For a pipeline of length 0.5 m and cross sectional area of 0.01 m^2 filled with liquid which flows with a volumetric flow $0.001 \text{ m}^3/\text{s}$, calculate the time-delay (transport delay) from inlet to outlet of the pipe.

6.5 Higher order systems

Exercise 6.7

Assume that a system can be well described by 3 time-constant systems in series, with the following time-constants respectively: 0.5, 1, and 2 sec. What is the approximate response time of the system?

Chapter 7

Feedback control

7.1 Introduction

No exercises here.

7.2 Function blocks in the control loop

Exercise 7.1

On Page 77 in the textbook you can read this statement: “If you do not have any other requirements for setting the filter time-constant, you can try setting it to 2 sec.” Assume that a filter is to be used on a tachometer¹ reading on an electrical motor which itself can be modelled as a time-constant system with time-constant of 0.2 sec. Is it wise to use 2 sec. as the filter time-constant?

Exercise 7.2

Given a temperature sensor which represents temperatures, T , in the range [15 – 55 °C] with a measurement signal (current), M , in the range [4 – 20 mA], with a linear relation between these ranges. Find the scaling function with C as input and T as output on the following form:

$$T = aM + b \tag{7.1}$$

¹A tachometer is a rotational speed sensor which gives a voltage being proportional to the speed.

Exercise 7.3

See Figure 7.5 in the text-book. Find the scaling function with u in the range $[0 - 3336 \text{ kg/min}]$ as input and u_1 in the range $[4 - 20 \text{ mA}]$ as output on the following form:

$$u_1 = au + b \quad (7.2)$$

What will you use this scaling function for in the level control system (on which the above mentioned figure is based)?

7.3 The PID controller**Exercise 7.4**

Suppose you want to verify that a PI controller works correctly according to the mathematical PI controller function. This can be done with a step response test, where a step is applied to the setpoint y_{SP} input to the controller while the measurement input y_m is kept constant. Assume that

$$y_m(t) = A \quad (7.3)$$

and that the setpoint is increased from A to

$$y_{SP}(t) = A + E \quad (7.4)$$

This implies that the control error e is increased as a step from zero to

$$e(t) = y_{SP}(t) - y_m(t) = (A + E) - A = E \quad (7.5)$$

By comparing the observed (experimental) step response in the controller output u with the theoretical output, you can (hopefully) confirm that the mathematical operation of the controller is correct.

The PI controller function is

$$u = K_p e + \frac{K_p}{T_i} \int_0^t e dt \quad (7.6)$$

1. Calculate the step response in u (as a function of time) assuming that the control error is a step of amplitude E , and plot $u(t)$.
2. Figure 7.1 shows the step response in u for a given PI controller. The step amplitude of the control error was

$$E = 1 \quad (7.7)$$

Calculate K_p and T_i from the step response.

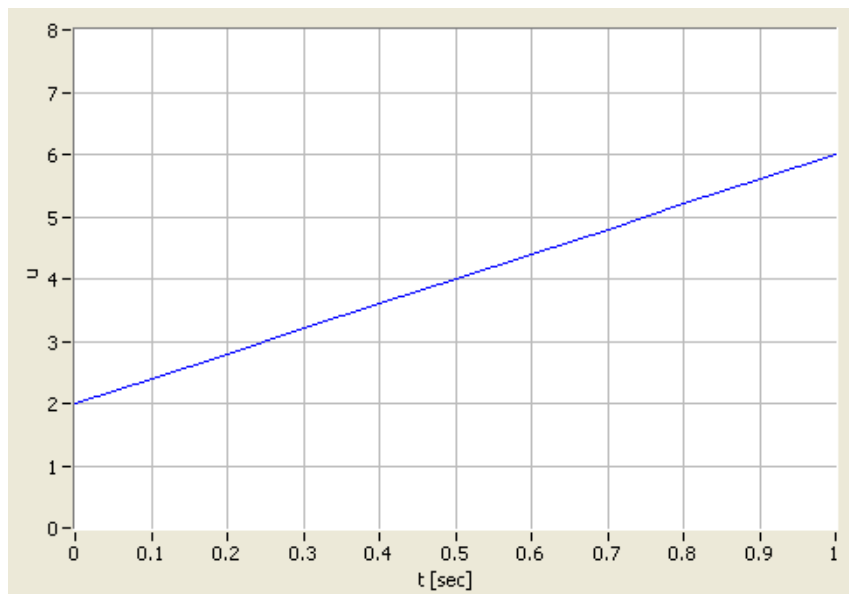


Figure 7.1:

Exercise 7.5

What is the value of the controller gain K_p corresponding to proportional band $PB = 250\%$?

Exercise 7.6

Figure 7.2 shows an air heater. A fan with fixed speed blows air through the pipe. The fan opening can be varied manually. The air is heated by an electrical heater. The control signal u is the voltage signal which controls (adjusts) the power supplied to the heater. The temperature is measured with a thermistor which is a temperature-dependent resistance. In the experiments described below the controller is implemented in a PC with I/O-device (Input/Output-device). (In general, a control system should contain a measurement filter, but in this particular system a filter was not used.)

1. Draw a block diagram of the control system, including a measurement filter.
2. Figure 7.3 shows the response in the temperature y after a step in the temperature reference (setpoint) and after a step in the air inflow

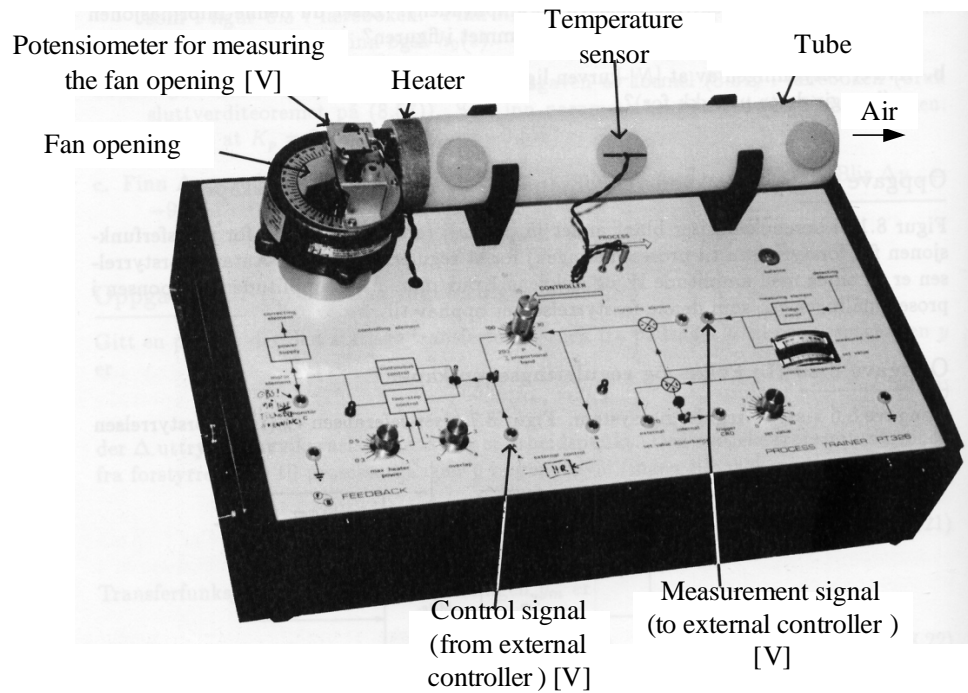


Figure 7.2:

due to an increase of the fan opening. The air inflow or – equivalently the fan opening – can be regarded as a disturbance. In this experiment the control is constant, hence there is no feedback (no measurement-based or error-driven) control.

Explain why there is no response in the temperature due to the reference change. And explain why there *is* a response after the disturbance (fan opening) change.

3. Assume that the temperature is controlled with a PID-controller (with proper parameter settings). Draw the principal temperature response after a reference step and after a disturbance step. You can “add” your curves to 7.3.)

Exercise 7.7

Figure 7.4 shows a pressure control system. Assume that increasing the control signal to the valve increases the valve opening. Will you set the controller to have reverse action or direct action?

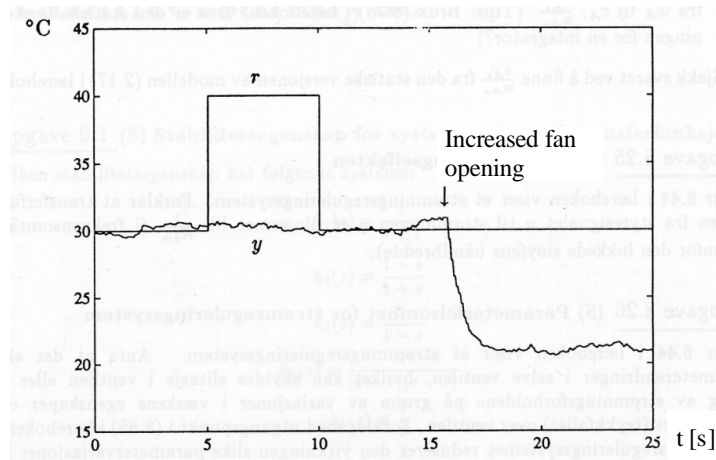


Figure 7.3:

7.4 Practical modifications of the ideal PID controller

7.4.1 Lowpass filter in the D-term

Exercise 7.8

Given a PID controller with a lowpass filter acting on the derivative term. Assume that the derivative time T_d is 2.0 sec. Suggest a proper value of the filter time-constant T_f .

7.4.2 Reducing P-kick and D-kick caused by setpoint changes

Exercise 7.9

A derivative term – or D-term – with the possibility of setpoint weight reduction is

$$u_d = K_p T_d \frac{d(w_d y_{SP} - y)}{dt} \quad (7.8)$$

(It is here, for simplicity, assumed that there is no lowpass filter acting on the D-term.) Assume that there is no reduced weight of the setpoint, i.e.

$$w_d = 1 \quad (7.9)$$

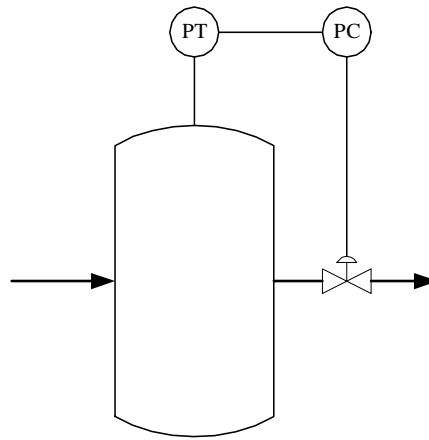


Figure 7.4:

Assume that the process measurement y is constant, and that the setpoint y_{SP} is changed as a step at time $t = 0$. Describe (qualitatively) the corresponding response in the control signal u_d due to this step. What kind of signal is this response?

7.4.3 Integrator anti wind-up

Exercise 7.10

One application where it is particularly important with anti windup is limiting control. Figure 7.5 shows a gas tank with one inlet and two outlets. The purpose of the normal control loop is to keep the gas pressure at the normal setpoint SP_1 , say 2 bar. The purpose of the limiting control loop is to limit the pressure to the higher setpoint SP_2 , say 4 bar.

What is the control error of controller PC_2 under normal conditions? Why is it particularly important that this controller has anti windup?

7.5 Control loop stability

Exercise 7.11

How will the stability of the control loop change if the proportional band PB is reduced?

Exercise 7.12

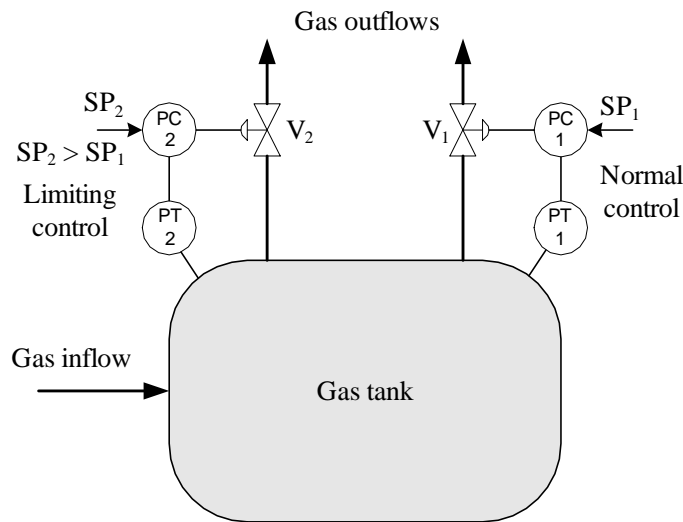


Figure 7.5:

Figure 7.6 shows a temperature control system with two different positions of the temperature sensor. Assume that the temperature controller is tuned so that the stability of the control system is satisfactory with the sensor in position 1.

1. What will happen to the stability if the sensor is moved to position 2?
2. Assume that the sensor is in position 1. How will the stability change if the liquid flow decreases?
3. How will the stability change if the sensor gain is increased (the gain is the ratio of the sensitivity of the measurement signal in volts or amperes to the temperature)?
4. How will the stability change if the heater is substituted by a heater delivering more power per unit of the control signal?

Exercise 7.13

A temperature control system is described in Exercise 7.6. Figure 7.7 shows the temperature response after a relatively large reduction of the fan opening. Explain the unstable response!

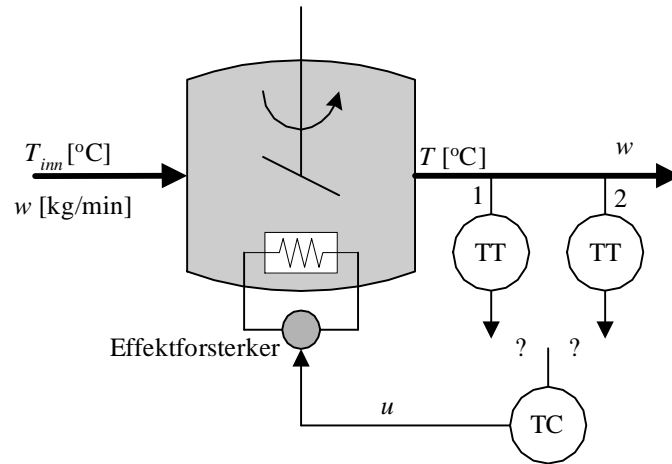


Figure 7.6: Temperature control system

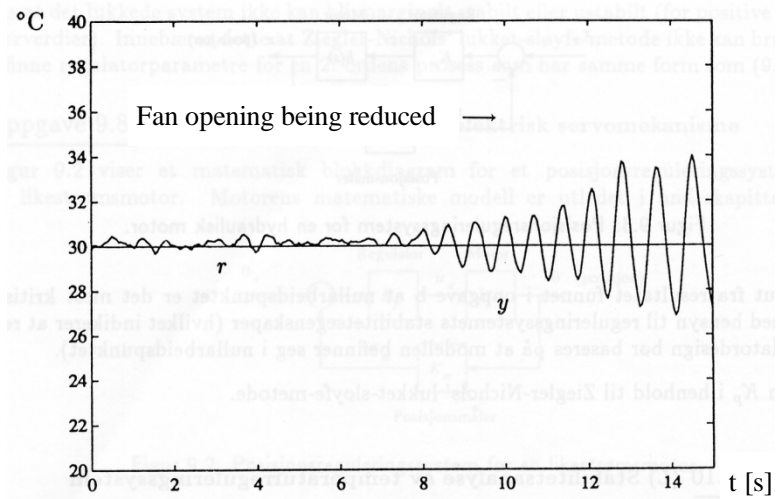


Figure 7.7:

Chapter 8

Feedforward control

8.1 Introduction

No exercises here.

8.2 Designing feedforward control from differential equation models

Exercise 8.1

See Exercise 3.4. The mathematical model of the ship is

$$\underline{\underline{m\ddot{x} = F_p - D|\dot{x} - u_c|(\dot{x} - u_c) + F_w}} \quad (8.1)$$

Assume that the positional reference is x_r [m]. F_p is the control variable. Design a feedforward controller for the ship. What information is needed to implement the feedforward controller? Is it realistic to get this information?

Exercise 8.2

Figure 8.1 shows a heated liquid tank where the temperature T shall be controlled using feedback with PID controller in combination with feedforward control. We assume the following process model, which is

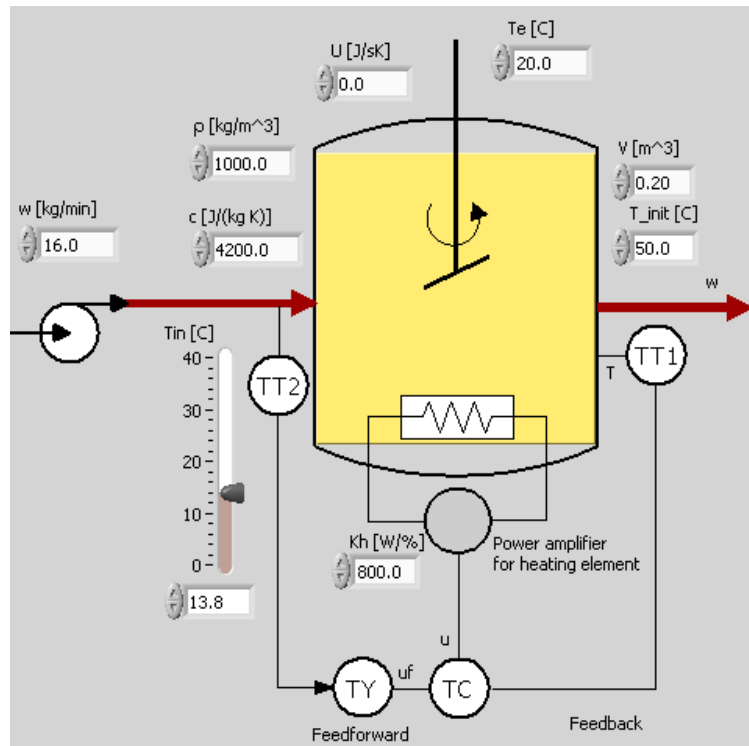


Figure 8.1:

based on energy balance:

$$c\rho V\dot{T}(t) = \underbrace{K_h u(t)}_P + cw [T_{in}(t) - T(t)] + U [T_e(t) - T(t)] \quad (8.2)$$

where T [K] is the temperature of the liquid in the tank, T_{in} [K] is the inlet temperature, T_e [K] is environmental temperature, c [J/(kg K)] is specific heat capacity, w [kg/s] is mass flow (same in as out), V [m³] is the liquid volume, ρ [kg/m³] is the liquid density, U [(J/s)/K] is the total heat transfer coefficient, $P = K_h u$ [J/min] is supplied power via heating element where K_h is a parameter (gain) and u [%] is the control signal applied to the heating element. $c\rho VT$ is the (temperature dependent) energy of the liquid in the tank. We can consider T_{in} and T_e as disturbances, but the derivation of the feedforward function F_f is not dependent of such a classification.

Derive the feedforward function from the process model (8.2). Which parameters and variables must have known values to implement the feedforward control?

8.3 Designing feedforward control from experimental data

Exercise 8.3

See Figure 8.7 in the text-book. Assume that you will implement the feedforward function as a set of linear functions (lines) between the data points (fan, u_{ff}) . The linear functions are on the form

$$u_{ff} = a \cdot fan + b \tag{8.3}$$

Let us select the points (40,52) and (60,61). Calculate a and b of the line between these two points.

Chapter 9

Controller equipment

Exercise 9.1

Given a controller with analog output (control signal) in the range 0 – 20 mA. Suppose the controller is to be used to control a device (e.g. a pump) that only takes voltage in the range 0 – 10 V. How can you (easily) convert the current control signal to a corresponding voltage control signal. Draw your solution.

Exercise 9.2

Where in the data sheet of the controller ECA600 shown in Figure 9.3 in the text-book is it shown that the controller has the options of reverse and direct action?

Exercise 9.3

For a certain biogas reactor the feed should have a value of 0.1 l/h (litres per hour). The feed pump delivers a maximum flow of 5 l/h. Lower pump flow rates than 0.5 l/h are very uncertain, so running the pump at such low flows may not give the flow you expect. To obtain such a small flow into the reactor as 0.1 l/h, the pump can be controlled with Pulse-Width Modulation (PWM) with maximum flow equal to 5 l/h and minimum flow equal to 0 l/h (pump is stopped). Assume that the hydraulic resident time (HRT) of the reactor, which is the same as the time-constant of the reactor, is 1 hour, and assume that the period of the PWM signal is 1 min, which is much smaller than 1 hour. This implies that the reactor will

“feel” the average value of the PWM flow, even if the flow is actually On-Off-On-Off etc.

What is the Duty Cycle, DC , of the pump flow in percent which gives the demanded flow of 0.1 l/h, and what is the corresponding On-time, T_{on} , of the pump flow?

Chapter 10

Tuning of PID controllers

10.1 Introduction

No exercises here.

10.2 The Good Gain method

Exercise 10.1

Figure 10.1 shows the response in the temperature of a simulated temperature control system with P controller with the following “good gain” value:

$$K_c = 4.0 \quad (10.1)$$

Tune a PI controller for this process using the Good Gain method.

What can you do with the controller tuning if turns out that the stability of the control system is too bad with this value of K_p ?

10.3 Skogestad’s PID tuning method

Exercise 10.2

Given a process where the relation between the control signal and the process measurement can be well represented as “time-constant with time-delay”:



Figure 10.1:

Gain:

$$K = 0.5 \quad (10.2)$$

Time-constant:

$$T = 5 \text{ s} \quad (10.3)$$

Time-delay:

$$\tau = 1 \text{ s} \quad (10.4)$$

Tune a PI(D) controller for this process using Skogestad's method.

Exercise 10.3

Figure 10.2 shows a level control system for a wood-chip tank with feed screw and conveyor belt which runs with constant speed. (This system is described in Section 7.2 of the text-book, however the measurement filter is omitted in the present example, assuming it has negligible effect on the dynamics of the control loop.) Based on mass balance of the wood-chip of the tank, a mathematical model is

$$\rho A \dot{y}(t) = K_s u(t - \tau) - w_{out}(t) \quad (10.5)$$

where ρ [kg/m³] is chip density, A [m²] is cross-sectional area, K_s [(kg/min)/%] is feed screw gain, τ [min] is time-delay of the conveyor belt. The level y is measured with a level sensor, and the measurement is

$$y_m(t) = K_m y(t) \quad (10.6)$$

where K_m [%/m] is the sensor gain.

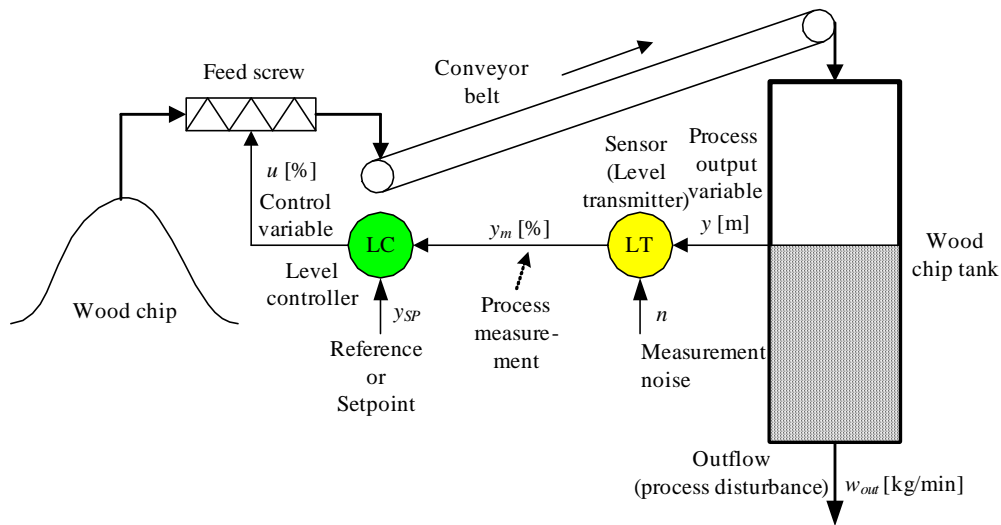


Figure 10.2:

1. Calculate the transfer function $H(s)$ from control variable u to process measurement y_m .
2. Calculate the PI(D) parameters for the process using Skogestad's method.

Exercise 10.4

Figure 10.3 shows the response in the filtered process measurement y_{m_f} due to a step of amplitude $U = 2$ in the control signal u (the step comes at time $t = 0$). Calculate PID settings for this process.

10.4 Auto-tuning

Exercise 10.5

1. In general, if you in a given application are to select between open loop excitation and closed loop excitation, which one would you select?
2. Are there any processes for which open loop excitation should not be used?

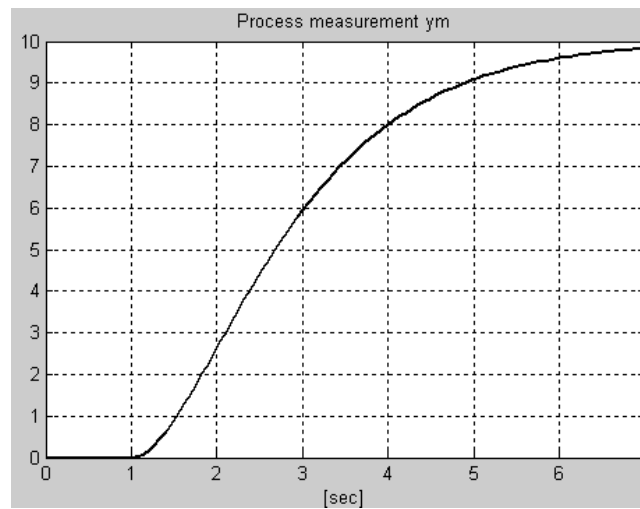


Figure 10.3:

10.5 PID tuning when process dynamics varies

Exercise 10.6

Assume that the process to be controlled has varying process dynamics, which may cause stability problems or sluggish control. Both of the solutions A and B below are possible. Which is the best one with respect to control performance, and which is the simplest one?

A: The controller is tuned to the most critical operating point, and the controller parameters are then kept constant.

B: The controller parameters are adjusted continually so that they fit to the dynamic properties of the process at any operating point.

Exercise 10.7

Assume that you in a given control system for a “time-constant with time-delay” process have found proper PI parameters in one specific operating point. Assume that the process gain increases.

1. How would this process gain increase influence the stability of the control system?

- Derive formulas for the new controller parameters. You can indicate the initial values of the controller parameters and the process parameters (before the change) with index 0, and new values (after the changes) with index 1.

Exercise 10.8

Figure 10.4 shows a chemical reactor and a PID parameter table which is the basis of a PID controller with gain scheduling. Assume that gain

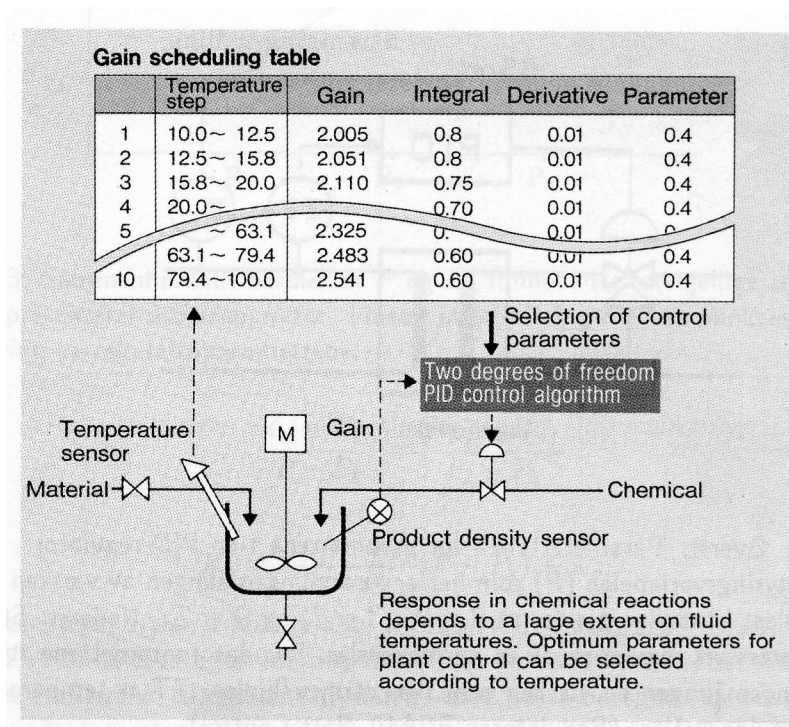


Figure 10.4:

scheduling is not to be used, but fixed PID settings in stead. Should the controller be tuned at high temperature or at low temperature, given that it is crucial that the stability of the control system is satisfactory at any temperature?

Exercise 10.9

Table 10.1 shows parts of a gain scheduling based PID controller.

GS	K_p	T_i	T_d
\vdots	\vdots	\vdots	\vdots
20%	0.4	5.2	1.3
30%	0.5	4.5	1.6
\vdots	\vdots	\vdots	\vdots

Table 10.1: PID Gain Schedule

Find K_p as a function of the gain scheduling variable GS between the operating points shown in the table. The function should be based on linear interpolation.

Exercise 10.10

Figure 10.5 shows a process with a PID control system where the actuator is represented with a nonlinear relation between the control signal u and the internal process variable z :

$$z = f(u) \quad (10.7)$$

For example, the actuator can be a control valve with some nonlinear relation between the control signal (u) and the flow (z). In most cases it is

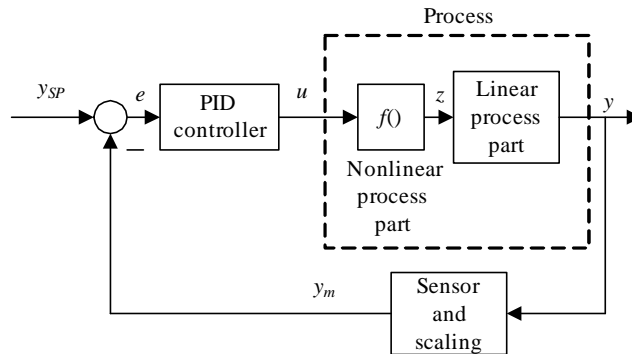


Figure 10.5:

beneficial if the PID controller sees a *linear* process – not a nonlinear process, because this makes the controller tuning easier, and the dynamic properties of the control system may be independent of the operating point. This can be achieved by including the *inverse* of the nonlinear function in the controller:

$$u = f^{-1}(z) \quad (10.8)$$

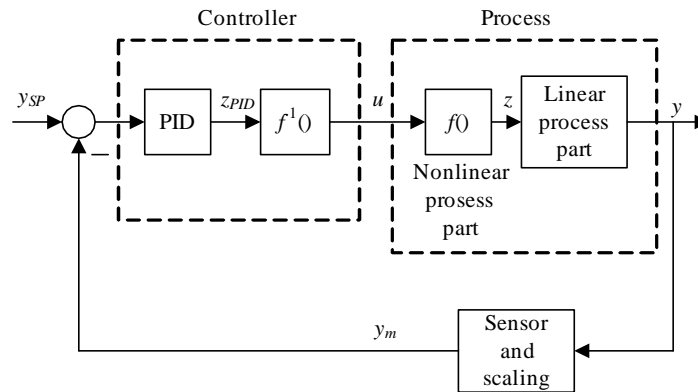


Figure 10.6:

The z -value that the PID controller demands can be denoted z_{PID} . See Figure 10.6.

Assume that the nonlinear function $z = f(u)$ can be represented with n tabular data points (which can stem from a data sheet or from experiments):

z	u
z_1	u_1
z_2	u_2
\vdots	\vdots
z_n	u_n

Explain how you can implement the inverse function using *table-lookup*. Table-lookup functions implements linear interpolation between the data points in the table.¹

¹Table lookup functions are available in computer tools as MATLAB and LabVIEW.

Chapter 11

Various control methods and control structures

11.1 Cascade control

Exercise 11.1

In the neutralization section of a fertilizer production plant, intermediate mother liquor flows into and out of a tank. In the tank the pH value of the liquid is controlled by adjusting the inflow of ammonia gas to the tank. The ammonia flow is flow controlled using a control valve.

Draw an instrumentation diagram of this process section. (You can use Q (for quality) as symbol for pH.) What can be the purpose of the ammonia flow control loop?

Exercise 11.2

Figure 11.1 shows a control valve being used to manipulate the flow of a heating medium (liquid) into a heat exchanger where the temperature is to be controlled. The output of the temperature controller is flow command signal (flow setpoint) to the valve, and the output of the flow controller is a valve stem position command (position setpoint) to the stem moving mechanism.

Draw an instrumentation diagram of the total control system. You can use symbol G for position of the valve stem. (Hint: There are three control loops.)

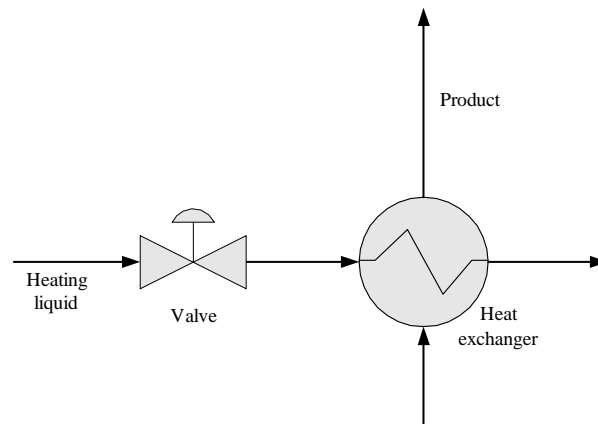


Figure 11.1:

Exercise 11.3

Figure 11.2 shows a ship. The position of the ship is controlled. Assume

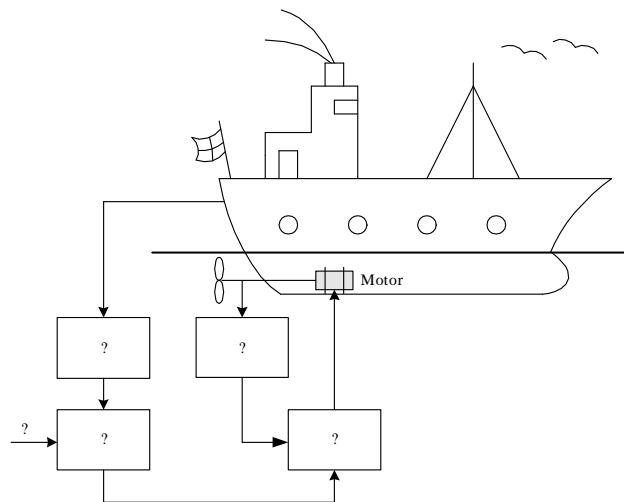


Figure 11.2:

that it is beneficial for the positional control system that the rotational speed of the propeller is controlled.

Based on the given information, substitute the question marks with proper functions (text). What are the purposes of the control loops?

11.2 Ratio control

Exercise 11.4

Figure 11.3 shows a tank with two inlet flows. The liquid level of the tank

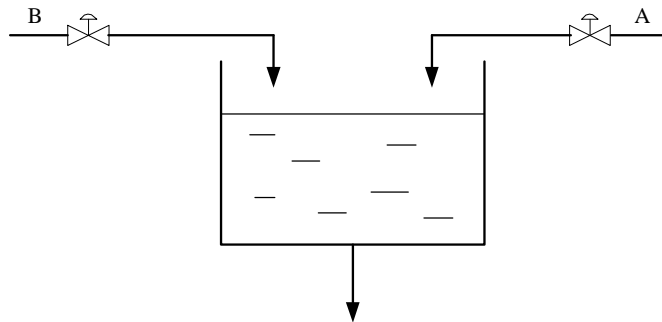


Figure 11.3:

is to be controlled by manipulating (controlling) flow A. It is assumed that flow A is much larger than flow B. The ratio between flow B and A is specified as

$$\frac{F_B}{F_A} = k \quad (11.1)$$

where k is a given ratio. Assume that it is necessary to have local flow control loops around each valve.

Draw a Process & Instrumentation diagram of a control system for this process.

11.3 Split-range control

Exercise 11.5

Figure 11.4 shows a liquid tank where the pH value of the liquid is to be controlled with split-range control where acid flow and base flow are adjusted. Both the acid flow and the base flow are controlled with (local) flow control loops. Draw an instrumentation diagram of the tank with control system.

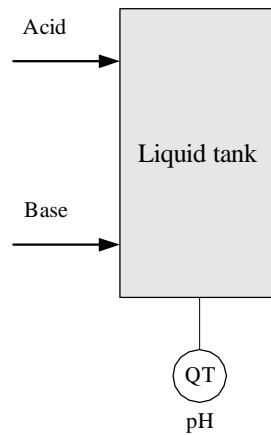


Figure 11.4:

11.4 Flow smoothing with sluggish level control

Exercise 11.6

See Figure 11.13 in the text-book. Assume that the process model is given by Eq. (11.4) in the text-book. The model is repeated here:

$$\rho A \dot{h} = F_{in} - F_{out} \quad (11.2)$$

Assume that the level is controller with a P controller:

$$F_{out} = K_p(h_{SP} - h) \quad (11.3)$$

where K_p is the controller gain. (The value of K_p will be negative because the controller must have direct action mode.)

1. Express the level h as a function of the setpoint h_{SP} and the inflow F_{in} under static conditions. (Hint: At static conditions the time-derivative is zero.) You can use index s for "static".
2. Assume that for a given flow, F_{in0} , the level are allowed to depart from the setpoint by Δh_s . (Thus, Δh_s is the level control error, e_s .) Calculate the corresponding controller gain.
Is Δh_s reduced or increased if K_p is increased?
3. What is the drawback, regarding the static control error, of using a P controller for level control of the tank?

11.5 Plantwide control

Exercise 11.7

Figure 11.5 shows a wood-chip tank, which is in the beginning of the pulp & paper production line. Spruce, pine and eucalyptus are used as feeds into the tank, via a conveyor belt. The percentages of each of these flows are indicated in Figure 11.5. There is a flow disturbance before the belt which is due to sieving the chip flow to remove large parts of chip.

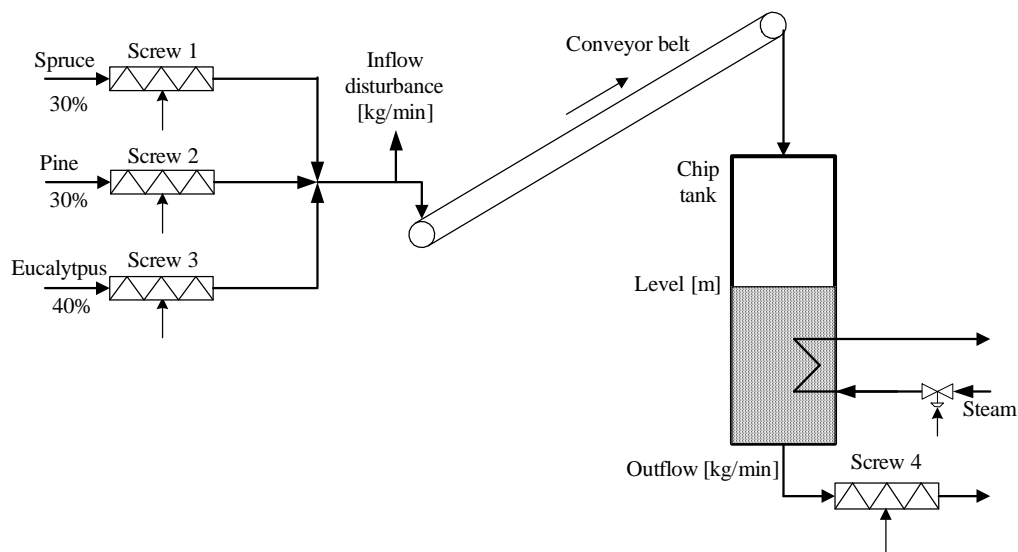


Figure 11.5:

Draw a P&I (Process & Instrumentation) diagram of a control system for this part of the production line according to the following specifications:

- The production rate is controlled to a setpoint with flow control of Screw 4.
- The level of chip in the tank is controlled to a setpoint by manipulating the total inflow to the conveyor belt.
- The total chip flow into the belt is splitted into percentage flows shown in Figure 11.5. The splitting can be represented with a block with total control signal (100%) as input and three flow value outputs (30%, 30% and 40% respectively). The flows out of the three inflow screws are flow controlled.

- A flow control loop is used to compensate for the flow disturbance due to the sieving. This flow loop is based on the measurement of the flow with a flow sensor at the beginning of the belt (cf. Figure 11.7 in the text-book).
- The temperature of the chip in the tank is controlled to a setpoint using the steam valve.

Exercise 11.8

Figure 11.6 shows an incomplete P&I (Process & Instrumentation) diagram of a controlled distillation column.

If you need it, here is some basic information about distillations columns: A distillation column contains a number of trays from where liquid can pour downwards (to the next tray) and vapour can rise upwards (to the next tray). The purpose of the distillation column is to separate the “light” component and the “heavy” component by exploiting their different boiling points of temperature. Heat is supplied to the boiler at the bottom of the column. Vapour leaving the column is condensed in the condenser. The liquid leaving the condenser is accumulated or stored in the accumulator. Part of the liquid leaving the accumulator is directed back to the column, and the rest – the distillation product – is directed to e.g. a storage tank. Ideally, the concentration of “heavy” component in the top product is zero, and the concentration of the “light” component in the bottoms product is zero. In principle this can be achieved by one quality control loop for the top product and one quality control loop for the bottoms product, but due to the dynamic properties of distillation columns such “two-point” control is difficult to realize. Therefore, there is typically either quality control of the top product or quality control of the bottoms product.

Make the diagram shown in Figure 11.6 complete by entering letter codes in the instrumentation symbols according to these specifications: The quality of the distillate product is controlled, and there is mass balance control of various parts of the column. (The heating medium supplied to the boiler is manually controlled, so it is not adjusted by an automatic controller.)

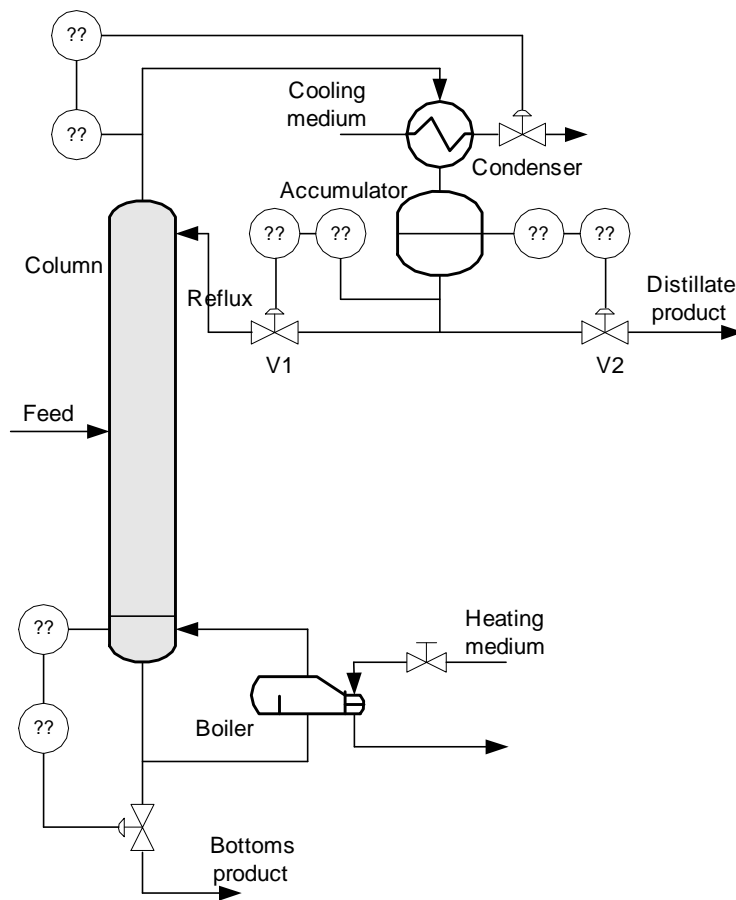


Figure 11.6:

Chapter 12

Sequential control

Exercise 12.1

Figure 12.1 shows a simple drilling machine. The machine operates as follows: The drilling operation is started with the Start button which sets

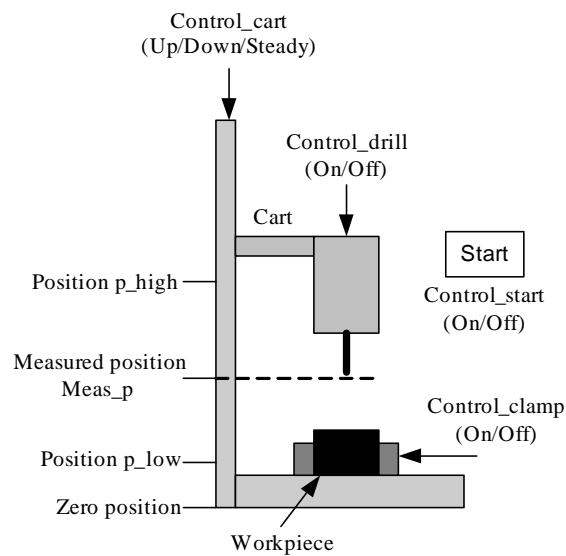


Figure 12.1:

the control signal `Control_start` to value `On`. Just after the the button has been pressed, it pops up automatically and `Control_start` is automatically set back to `Off` (this reset is not a part of the control task in this exercise). When the drilling operation has been started, the clamps are activated by

setting the control signal `Control_clamp` to `On`, the drill starts rotating with `Control_drill` set to `On`, and the cart is moved downwards with `Control_cart` set to `Down` until the measured drill position `Meas_p` becomes `p_low`. Then, the cart is automatically moved upwards with `Control_cart` set to `Up`. When the `Meas_p` has become `p_high`, the cart is stopped with `Control_cart` set to `Steady`, the clamp is released with `Control_clamp` set to `Off`, and the drill is stopped with `Control_drill` set to `Off`. Then the drill is idle, waiting until the `Start` button is again pressed.

Placing the workpiece in the correct position is not a part of this control task.

Draw a Sequential Function Chart (SFC) with steps, actions and transitions solving the control task given above.

Part II

SOLUTIONS

Solution to Exercise 1.5

Examples are given in the first chapter of the text-book (no additional examples are given here).

Solution to Exercise 1.1

1. Figure 12.2 shows the speed control system. The control error, e , is

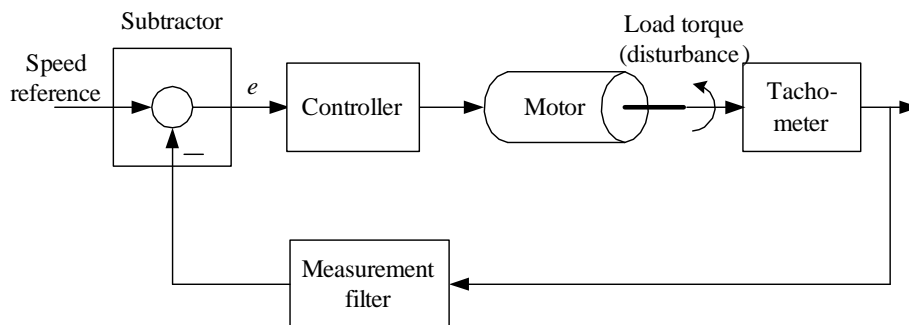


Figure 12.2:

the output of the subtractor.

2. When the speed is reduced due to the increased load torque, the control error becomes different from zero, and positive. This non-zero, positive control error causes the controller to increase the control signal acting on the motor, so that the speed is increased. If the controller is properly chosen (it must have integral action, actually), the controller is able to adjust the control signal to exactly the new value that is needed to compensate for the load torque, and consequently the control error become zero – in steady state.

Solution to Exercise 1.2

The purpose of the system is to fill just the right amount of liquid into the cup, i.e. level control. The system works as follows: As long as the level is low the inlet is open and the cup is being filled. The more liquid in the cup, the less opening in the inlet. Eventually, when the cup is full, the inlet is closed and hence, the desired level (the level setpoint) is reached.

Another system: Water toilet.

Solution to Exercise 1.3

The control structure is shown in Figure 12.3.

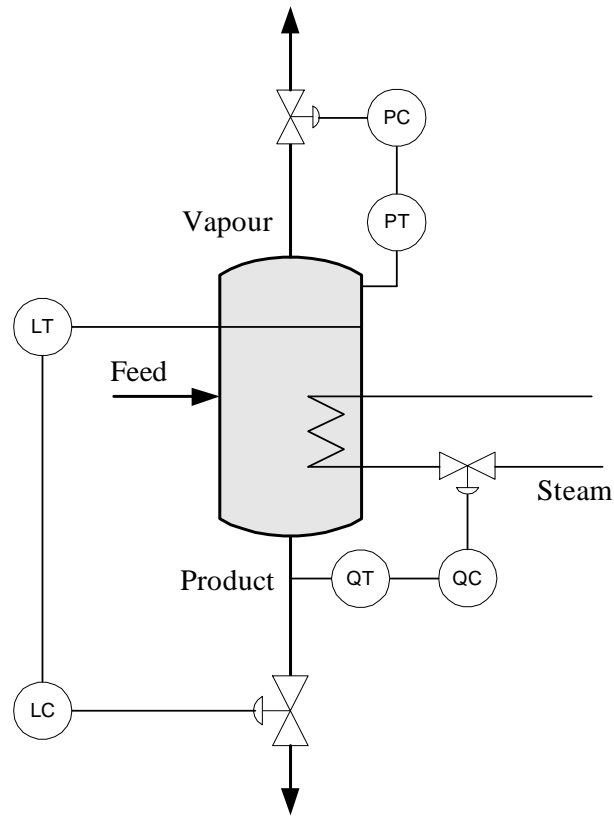


Figure 12.3:

Solution to Exercise 1.4

1. Robot arm: Control signal manipulates the motor. Disturbances: Torques due to the gravity and due to mechanical couplings to other arms.
2. Heat exchanger: Control signal manipulates the valve. Disturbances: Temperature and pressure of inlet steam.
3. Ship: Control signal manipulates the propellers (thrusters). Disturbances: Wind, current, waves.

Solution to Exercise 2.1

Three examples of dynamic systems:

1. System: Airplane: Inputs: Control signal to motors and flaps. Output: Airplane positions (altitude, pitch, yaw).
2. System: Chemical reactor. Input: Feed flow. Output: Concentration of the contents of the reactor.
3. System: Bank account: Inputs: Salary paid into the account, and withdrawal. Output: Amount of money in the account.

Solution to Exercise 2.2

Before drawing the block diagram, it is convenient to write the model with the time-derivative isolated at the left hand side:

$$\dot{h}(t) = \frac{1}{A} \left[K_u u(t) - K_v \sqrt{\rho g h(t)} \right] \quad (12.1)$$

1. See Figure 12.4.

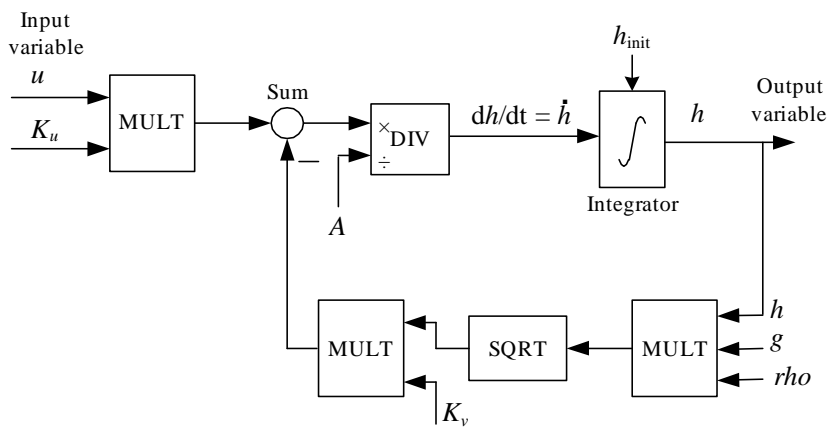


Figure 12.4:

2. See Figure 12.5.
3. One use of such a block diagram is for creating a simulator in a block diagram tool as LabVIEW Simulation Module or Simulink.

Solution to Exercise 2.3

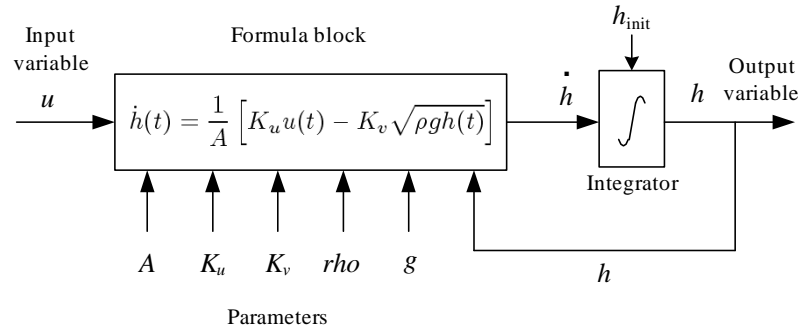


Figure 12.5:

Density ρ can be cancelled. (2.2) becomes

$$\underline{\underline{\dot{h}_1 = \frac{1}{A_1} \left(\overbrace{K_p u_1 - K_{v1} \sqrt{\frac{\rho g h_1}{G}}}^{f_1(\cdot)} \right)}} \quad (12.2)$$

(2.3) becomes

$$\underline{\underline{\dot{h}_2 = \frac{1}{A_2} \left(\overbrace{K_{v1} \sqrt{\frac{\rho g h_1}{G}} - K_{v2} u_2 \sqrt{\frac{\rho g h_2}{G}}}^{f_2(\cdot)} \right)}} \quad (12.3)$$

The measurement equations become

$$\underline{\underline{y_1 = \overbrace{h_1}^{g_1(\cdot)}}}} \quad (12.4)$$

$$\underline{\underline{y_2 = \overbrace{h_2}^{g_2(\cdot)}}}} \quad (12.5)$$

Solution to Exercise 2.4

The time-derivative in (2.1) is set to zero:

$$A \dot{h}_s(t) = 0 = K_u u_s - K_v \sqrt{\rho g h_s} \quad (12.6)$$

Solving for u_s gives

$$\underline{\underline{u_s = \frac{K_v \sqrt{\rho g h_s}}{K_u}}} \quad (12.7)$$

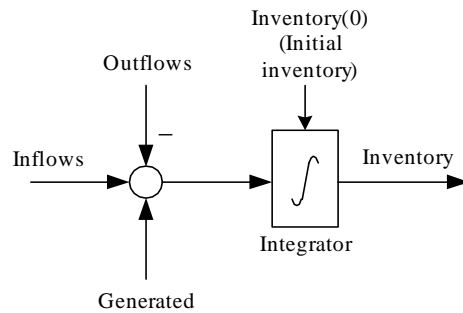


Figure 12.6:

Solution to Exercise 3.1

See Figure 12.6.

Solution to Exercise 3.2

1. Since there is a time delay in the system (due to the transport delay of the conveyor belt) it is important to include the time argument in the equations. The mass balance if the chip contents of the tank is

$$\begin{aligned}
 \frac{d}{dt} [\rho Ah(t)] &= \underline{\underline{\rho A \dot{h}(t)}} = w_{in}(t) - w_{out}(t) \\
 &= w_s(t - \tau) - w_{out}(t) \\
 &= \underline{\underline{K_s u(t - \tau) - w_{out}(t)}}
 \end{aligned}
 \tag{12.8}$$

2. Figure 12.7 shows the overall block diagram.

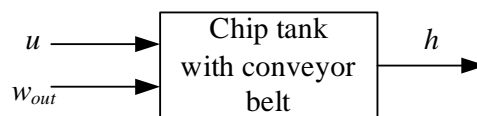


Figure 12.7:

3. Figure 12.8 shows the mathematical block diagram (it can be drawn in many other ways).

Solution to Exercise 3.3

Energy balance of the liquid in the tank:

$$\frac{d(c\rho VT)}{dt} = c\rho V\dot{T} = c\rho q_k T_k + c\rho q_v T_v - c\rho q T
 \tag{12.9}$$

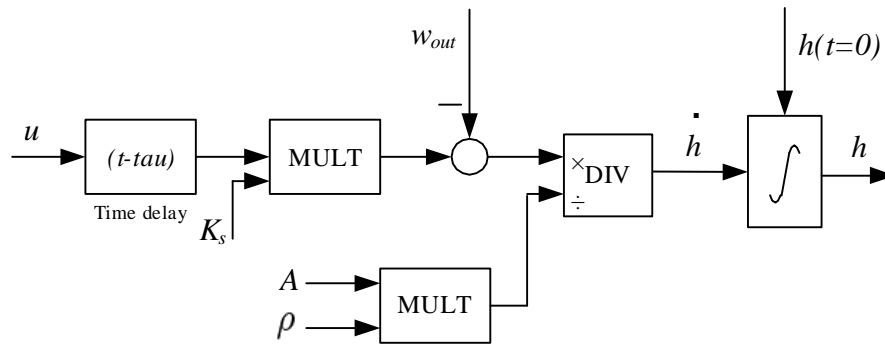


Figure 12.8:

Cancelling ρ :

$$\underline{\underline{cV\dot{T} = cq_k T_k + cq_v T_v - cqT}} \quad (12.10)$$

Here, q is given by

$$\underline{\underline{q = q_k + q_v}} \quad (12.11)$$

Solution to Exercise 3.4

1. The relation between position x and speed u is

$$\underline{\underline{\dot{x} = u}} \quad (12.12)$$

2. Force balance:

$$\underline{\underline{m\dot{u}}} = F_p + F_h + F_w \quad (12.13)$$

$$= \underline{\underline{F_p - D|u - u_c|(u - u_c) + F_w}} \quad (12.14)$$

(12.12) and (12.14) constitutes the model.

Alternatively, since

$$\dot{u} = \ddot{x} \quad (12.15)$$

the model can be expressed as

$$\underline{\underline{m\ddot{x} = F_p - D|\dot{x} - u_c|(\dot{x} - u_c) + F_w}} \quad (12.16)$$

3. We can regard F_p , F_w and u_c as input variables, and x as the output variable. Figure 12.9 shows the block diagram.

Solution to Exercise 3.5

Torque balance:

$$\underline{\underline{J\ddot{\theta} = T}} \quad (12.17)$$

Solution to Exercise 3.6



Figure 12.9:

1. Force balance (Newton's Second Law) applied to the horizontal movement of the center of gravity of the pendulum.

$$\underline{\underline{m \frac{d^2}{dt^2} (y + L \sin a) = H}} \quad (12.18)$$

(The differentiation of the additive term $(y + L \sin a)$ must be carried out in applications of this model, but it is not shown here.)

2. Force balance applied to the vertical movement of the center of gravity of the pendulum:

$$\underline{\underline{m \frac{d^2}{dt^2} (L \cos a) = V - mg}} \quad (12.19)$$

(The differentiation of the additive term $(L \cos a)$ is not shown here.)

3. Torque balance (the rotational version of the Newton's Second Law applied to the center of gravity of the pendulum:

$$\underline{\underline{I\ddot{a} = VL \sin a - HL \cos a}} \quad (12.20)$$

4. Force balance applied to the cart:

$$\underline{\underline{M\ddot{y} = F - H - d\dot{y}}} \quad (12.21)$$

(From Eq. (12.18) – (12.21), the internal forces V and H can be eliminated, resulting in two differential equations not containing V and H .)

Solution to Exercise 3.7

The circuit consists of two resistors in parallel in series with the third resistor. The resulting resistance is

$$\underline{\underline{R_4 = \frac{R_1 R_2}{R_1 + R_2} + R_3}} \quad (12.22)$$

Solution 3.8

Mean power is

$$P = \frac{1}{2} \frac{U^2}{R} \quad (12.23)$$

which solved for R gives

$$\underline{\underline{R}} = \frac{1}{2} \frac{U^2}{P} = \frac{1}{2} \frac{220^2}{100} = \underline{\underline{242 \Omega}} \quad (12.24)$$

Solution to Exercise 3.9

There are many ways to find a mathematical model. Here is one:
Kirchhoff's voltage law gives

$$-v_1 + v_C + v_2 = 0 \quad (12.25)$$

or

$$v_C = v_1 - v_2 \quad (12.26)$$

Kirchhoff's current law applied to the upper node gives

$$0 = i_C - i_R + \overbrace{i_2}^{=0} \quad (12.27)$$

$$= C \frac{dv_C}{dt} - \frac{v_2}{R} \quad (12.28)$$

$$= C \frac{d(v_1 - v_2)}{dt} - \frac{v_2}{R} \quad (12.29)$$

$$= C(\dot{v}_1 - \dot{v}_2) - \frac{v_2}{R} \quad (12.30)$$

which can be written as

$$\underline{\underline{RC\dot{v}_2 - v_2 = RC\dot{v}_1}} \quad (12.31)$$

Solution to Exercise 4.1

We set $f(t) = e^{-t}$ in the integral that defines the Laplace transform:

$$\begin{aligned} \underline{\underline{\mathcal{L}\{e^{-t}\}}} &= \int_0^{\infty} e^{-st} e^{-t} dt \\ &= \int_0^{\infty} e^{-(s+1)t} dt \\ &= \frac{1}{-(s+1)} \left[e^{-(s+1)t} \right]_{t=0}^{t=\infty} \\ &= \frac{1}{-(s+1)} [0 - 1] \\ &= \underline{\underline{\frac{1}{s+1}}} \end{aligned}$$

The proper Laplace transform pair is:

$$\frac{k}{Ts + 1} \iff \frac{ke^{-t/T}}{T} = e^{-t} \quad (12.32)$$

Here, $T = 1$ and $k = 1$. Thus, $F(s)$ becomes

$$\underline{\underline{F(s) = \frac{1}{s+1} = \mathcal{L}\{e^{-t}\}}} \quad (12.33)$$

which is the same as found above using the definition of the Laplace transform.

Solution to Exercise 4.2

1. To calculate $y(t)$ we start by taking the Laplace transform of both sides of the given differential equation:

$$\mathcal{L}\{\dot{y}(t)\} = \mathcal{L}\{-2y(t) + u(t)\} \quad (12.34)$$

Here, we apply the time derivative property, cf. Eq. (4.10) in the text-book, at the left side, and the linear combination property, cf. Eq. (4.14) in the text-book, to the right side, to get

$$sY(s) - 4 = -2Y(s) + U(s) \quad (12.35)$$

Here,

$$U(s) = \frac{1}{s} \quad (12.36)$$

since the Laplace transform of a step of amplitude 1 is $\frac{1}{s}$, cf. transform pair (4.7) in the text-book.

By now we have

$$sY(s) - 4 = -2Y(s) + \frac{1}{s} \quad (12.37)$$

Solving for $Y(s)$ gives

$$Y(s) = \underbrace{\frac{4}{s+2}}_{Y_1(s)} + \underbrace{\frac{1}{(s+2)s}}_{Y_2(s)} \quad (12.38)$$

To get the corresponding $y(t)$ from this $Y(s)$ we take the inverse Laplace transform of $Y_1(s)$ and $Y_2(s)$ to get $y_1(t)$ and $y_2(t)$ respectively, and then we calculate $y(t)$ as

$$y(t) = y_1(t) + y_2(t) \quad (12.39)$$

according to the linearity property of the Laplace transform. $y_1(t)$ and $y_2(t)$ are calculated below.

Calculation of $y_1(t)$:

We can use the transform pair (4.10) in the text-book, which is repeated here:

$$\frac{k}{Ts + 1} \iff \frac{ke^{-t/T}}{T} \quad (12.40)$$

We have

$$Y_1(s) = \frac{4}{s + 2} = \frac{2}{0.5s + 1} \quad (12.41)$$

Hence, $k = 2$, and $T = 0.5$. Therefore,

$$y_1(t) = \frac{ke^{-t/T}}{T} = \frac{2e^{-t/0.5}}{0.5} = 4e^{-2t} \quad (12.42)$$

Calculation of $y_2(t)$:

We can use the transform pair (4.11) in the text-book, which is repeated here:

$$\frac{k}{(Ts + 1)s} \iff k(1 - e^{-t/T}) \quad (12.43)$$

We have

$$Y_2(s) = \frac{1}{(s + 2)s} = \frac{0.5}{(0.5s + 1)s} \quad (12.44)$$

Hence, $k = 0.5$, and $T = 0.5$. Therefore,

$$y_2(t) = k(1 - e^{-t/T}) = 0.5(1 - e^{-t/0.5}) = 0.5(1 - e^{-2t}) \quad (12.45)$$

The final result becomes

$$\underline{y(t)} = y_1(t) + y_2(t) \quad (12.46)$$

$$= 4e^{-2t} + 0.5(1 - e^{-2t}) \quad (12.47)$$

$$= \underline{\underline{0.5 + 3.5e^{-2t}}} \quad (12.48)$$

2. Using the Final Value Theorem on (12.38):

$$\underline{y_s} = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} s \left[\frac{4}{s + 2} + \frac{1}{(s + 2)s} \right] \quad (12.49)$$

$$= \lim_{s \rightarrow 0} s \frac{4}{s + 2} + \lim_{s \rightarrow 0} s \frac{1}{(s + 2)s} = 0 + \frac{1}{2} = \underline{\underline{0.5}} \quad (12.50)$$

From (12.48) we get

$$y_s = \lim_{t \rightarrow \infty} y(t) = 0.5 \quad (12.51)$$

And from the differential equation we get (because the time-derivative is zero in steady-state)

$$0 = -2y_s(t) + u_s(t) \quad (12.52)$$

which gives

$$y_s = \frac{u_s}{2} = \frac{1}{2} = 0.5 \quad (12.53)$$

So, the three results are the same.

Solution to Exercise 5.1

The Laplace transform of (5.1) is

$$\rho A [sh(s) - h_0] = K_s e^{-\tau s} u(s) - w_{out}(s) \quad (12.54)$$

Solving for output variable h gives

$$h(s) = \frac{1}{s} h_0 + \underbrace{\frac{K_s}{\rho A s} e^{-\tau s} u(s)}_{H_1(s)} + \underbrace{\left(-\frac{1}{\rho A s}\right) w_{out}(s)}_{H_2(s)} \quad (12.55)$$

Thus, the transfer functions are

$$\underline{\underline{H_1(s) = \frac{K_s}{\rho A s} e^{-\tau s}}} \quad (12.56)$$

and

$$\underline{\underline{H_2(s) = -\frac{1}{\rho A s}}} \quad (12.57)$$

Solution to Exercise 5.2

Laplace transform of (5.2) gives

$$m [s^2 y(s) - s \dot{y}_0 - y_0] = F(s) - D [s y(s) - y_0] - K y(s) \quad (12.58)$$

Setting initial values $y_0 = 0$ and $\dot{y}_0 = 0$, and then solving for $y(s)$ gives

$$y(s) = \frac{1}{\underbrace{ms^2 + Ds + K}_{H(s)}} F(s) \quad (12.59)$$

The transfer function is

$$\underline{\underline{H(s) = \frac{y(s)}{F(s)} = \frac{1}{ms^2 + Ds + K}}} \quad (12.60)$$

Solution to Exercise 5.3

1. Order: 2.
2. $s^2 + 3s + 2 = 0$
3. $s^2 + 3s + 2$
4. We write the transfer function on pole-zero-form:

$$H(s) = \frac{s + 3}{s^2 + 3s + 2} = \frac{s + 3}{(s + 1)(s + 2)} \quad (12.61)$$

We see that the poles are -1 and -2 , and the zero is -3 .

Solution to Exercise 5.4

1. Figure 12.10 shows the block diagram.

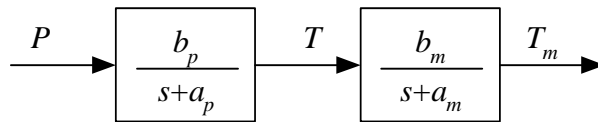


Figure 12.10:

2. According to the *series combination rule* the transfer function becomes

$$\underline{\underline{H(s) = \frac{T_m(s)}{P(s)} = H_m(s)H_p(s) = \frac{b_m}{s + a_m} \frac{b_p}{s + a_p}}} \quad (12.62)$$

Solution to Exercise 5.5

The Laplace transform of $u(t)$ is (cf. Eq. (4.7) in the text-book)

$$u(s) = \frac{3}{s} \quad (12.63)$$

Inserting this into (5.6) gives

$$y(s) = \frac{5}{s} \cdot \frac{3}{s} = \frac{15}{s^2} \quad (12.64)$$

which has the same form as in the Laplace transform pair given by Eq. (4.8) in the text-book. This transform pair is repeated here:

$$\frac{k}{s^2} \iff kt \quad (12.65)$$

We have $k = 15$, so the response is

$$\underline{\underline{y(t) = 15t}} \quad (12.66)$$

Solution to Exercise 5.6

Setting $s = 0$ in the transfer function gives

$$\underline{\underline{H_s}} = H(0) = \frac{1}{\underline{\underline{K}}} \quad (12.67)$$

The static response y_s corresponding to a constant force, F_s , is

$$\underline{\underline{y_s}} = H_s F_s = \frac{F_s}{\underline{\underline{K}}} \quad (12.68)$$

Solution to Exercise 6.1

1. Yes! Because the transfer function has the form of K_i/s .
2. The Laplace transform of the response is

$$h(s) = H_2(s)w_{out}(s) = -\frac{1}{\rho A s} w_{out}(s) \quad (12.69)$$

Since $w_{out}(t)$ is a step of amplitude W at $t = 0$, $w_{out}(s)$ becomes (cf. Eq. (4.7) in the text-book)

$$w_{out}(s) = \frac{W}{s} \quad (12.70)$$

With this $w_{out}(s)$ (12.69) becomes

$$h(s) = -\frac{1}{\rho A s} \frac{W}{s} \quad (12.71)$$

According to Eq. (4.8) in the text-book),

$$h(t) = -\frac{W}{\rho A} t \quad (12.72)$$

That is, the response is a ramp with negative slope.

Comment: This $h(t)$ is only the *contribution* from the outflow to the level. To calculate the complete response in the level, the total model (5.1), where both u and w_{out} are independent or input variables, must be used.

Solution to Exercise 6.2

Energy balance:

$$c\rho V \frac{dT}{dt} = P \quad (12.73)$$

Laplace transformation:

$$c\rho V [sT(s) - T_0] = P(s) \quad (12.74)$$

which yields

$$T(s) = \frac{1}{s}T_0 + \underbrace{\frac{1}{c\rho V s}}_{H(s)}P(s) \quad (12.75)$$

The transfer function is

$$H(s) = \frac{T(s)}{P(s)} = \frac{1}{c\rho V s} = \frac{K}{s} \quad (12.76)$$

which is the transfer function of an integrator with gain $K = 1/c\rho V$.

Solution to Exercise 6.3

We manipulate the transfer function so that the constant term of the denominator is 1:

$$H(s) = \frac{2}{4s + 8} = \frac{2/8}{(4/8)s + 8/8} = \frac{0.25}{0.5s + 1} = \frac{K}{Ts + 1} \quad (12.77)$$

Hence,

$$\underline{\underline{K = 0.25; \quad T = 0.5}} \quad (12.78)$$

We base the drawing of the step response on the following information:

- The steady-state value of the step response:

$$y_s = KU = 0.25 \cdot 6 = 1.5 \quad (12.79)$$

- The time-constant:

$$T = 0.5 \quad (12.80)$$

which is the time when the step response has reached value

$$0.63 \cdot y_s = 0.63 \cdot 1.5 = 0.95 \quad (12.81)$$

- The initial slope of the step response:

$$S_0 = \dot{y}(0^+) = \frac{KU}{T} = \frac{0.25 \cdot 6}{0.5} = 3 \quad (12.82)$$

Figure 12.11 shows the step response.

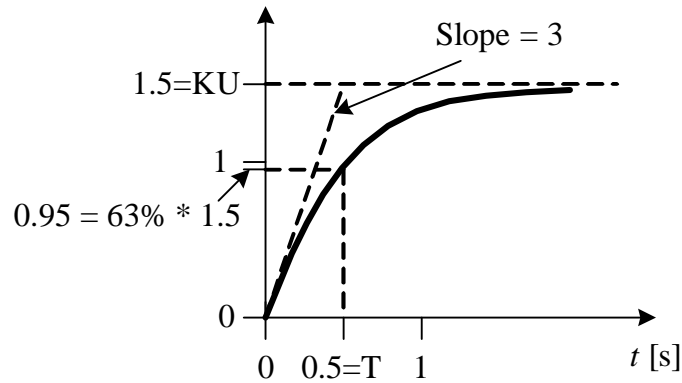


Figure 12.11:

Solution to Exercise 6.4

From Figure 6.2 we see that the gain is

$$K = \frac{\Delta T}{\Delta P} = \frac{30 \text{ K} - 20 \text{ K}}{1 \text{ kW}} = 10 \frac{\text{K}}{\text{kW}} \quad (12.83)$$

and that the time constant (the 63% rise time) is

$$T_1 = 50 \text{ min} \quad (12.84)$$

The transfer function becomes

$$\underline{\underline{\frac{\Delta T(s)}{\Delta P(s)} = \frac{10 \text{ K}}{50s + 1 \text{ kW}}}} \quad (12.85)$$

Solution to Exercise 6.5

1. Laplace transformation of the differential equation (6.4) gives

$$RCsv_{in}(s) = v_{out}(s) - v_{in}(s) \quad (12.86)$$

Solving for $v_{out}(s)$ gives

$$v_{out}(s) = \frac{1}{RCs + 1} v_{in}(s) \quad (12.87)$$

The transfer function is

$$\underline{\underline{H(s) = \frac{1}{RCs + 1} = \frac{K}{Ts + 1}}} \quad (12.88)$$

The gain is

$$\underline{\underline{K = 1}} \quad (12.89)$$

The time-constant is

$$\underline{\underline{T = RC}} \quad (12.90)$$

2. The filtering is stronger if R is increased.

Solution to Exercise 6.6

The time-delay is

$$\underline{\underline{\tau = \frac{AL}{q} = \frac{0.01 \text{ m}^2 \cdot 0.5 \text{ m}}{0.001 \text{ m}^3/\text{s}} = \underline{\underline{5 \text{ s}}}}} \quad (12.91)$$

Solution to Exercise 6.7

The approximate response time is

$$\underline{\underline{T = 0.5 + 1 + 2 = \underline{\underline{3.5 \text{ s}}}}} \quad (12.92)$$

Solution to Exercise 7.1

No! Because the filter will be much more sluggish than the motor. Probably, real speed information will be filtered or smoothed out (depending on how quick the real speed actually varies).

Solution to Exercise 7.2

The slope a can be calculated from

$$\underline{\underline{a = \frac{T_{\max} - T_{\min}}{M_{\max} - M_{\min}} = \frac{55 - 15}{20 - 4} = \frac{40}{16} = 2.5 \frac{\text{°C}}{\text{mA}}}} \quad (12.93)$$

and

$$\underline{\underline{\underline{\underline{b}}}} = T_{\min} - aM_{\min} = 15 \text{ °C} - 2.5 \frac{\text{°C}}{\text{mA}} \cdot 4 \text{ mA} = \underline{\underline{5 \text{ °C}}} \quad (12.94)$$

Solution to Exercise 7.3

The slope a can be calculated from

$$\underline{a} = \frac{u_{1 \max} - u_{1 \min}}{u_{\max} - u_{\min}} = \frac{20 - 4}{3336 - 0} = \frac{16}{3336} \frac{\text{mA}}{\text{kg/min}} = \underline{\underline{0.0048 \frac{\text{mA}}{\text{kg/min}}}} \quad (12.95)$$

and

$$\underline{b} = u_{1 \min} - au_{\min} = 4 \text{ mA} - \frac{16}{3336} \frac{\text{mA}}{\text{kg/min}} \cdot 0 \frac{\text{kg}}{\text{min}} = \underline{\underline{4 \text{ mA}}} \quad (12.96)$$

The scaling function $u_1 = au + b$ is used to transform the flow value in kg/min demanded by the level controller (as the controller output signal) to a corresponding current signal in mA to be applied to the feed screw.

Solution 7.4

1. Setting $e = E$ in the controller function gives

$$\underline{\underline{u(t)}} = K_p E + \frac{K_p}{T_i} \int_0^t E dt = \underline{\underline{K_p E + \frac{K_p}{T_i} E t}} \quad (12.97)$$

which is “constant plus ramp”. Figure 12.12 shows this step response.

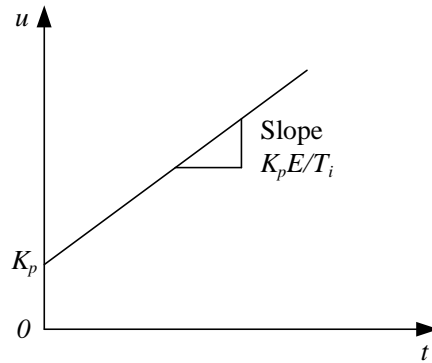


Figure 12.12:

2. From Figure 7.1 we see that

$$u(0_+) = \underline{\underline{2 = K_p}} \quad (12.98)$$

And we see that the slope is

$$\text{Slope} = 4 = \frac{K_p}{T_i} E = \frac{2}{T_i} \cdot 1 \quad (12.99)$$

which gives

$$\underline{\underline{T_i = 0.5 \text{ sec}}} \quad (12.100)$$

Solution to Exercise 7.5

$$\underline{\underline{K_p}} = \frac{100}{\text{PB}} = \frac{100}{250} = \underline{\underline{0.4}} \quad (12.101)$$

Solution to Exercise 7.6

1. The block diagram is shown in Figure 12.13.

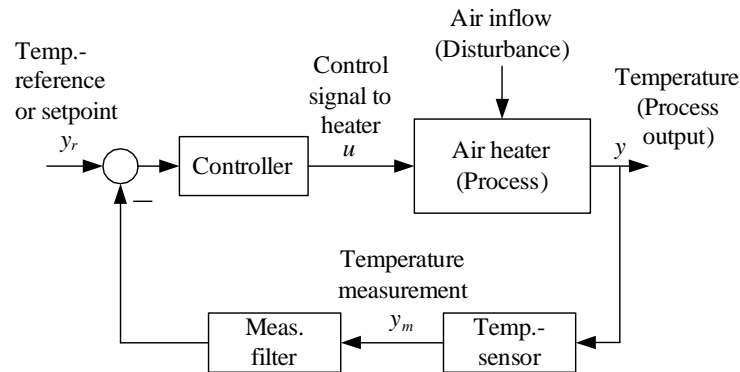


Figure 12.13:

2. Since u_0 is not influenced by the temperature reference (setpoint), the control signal remains constant. Therefore, the reference does not influence the actual temperature. However, an increase of the the fan opening will influence (reduce) the temperature because more cold air is blown into the pipe.
3. The PID-controller gives zero control error (in average), see Figure 12.14.

Solution to Exercise 7.7

Assume that for some reason the pressure is larger than the pressure reference. The controller must react to this by ensuring that the valve opening is increased, which means that the valve control signal is increased. Therefore, the controller must have directe action.

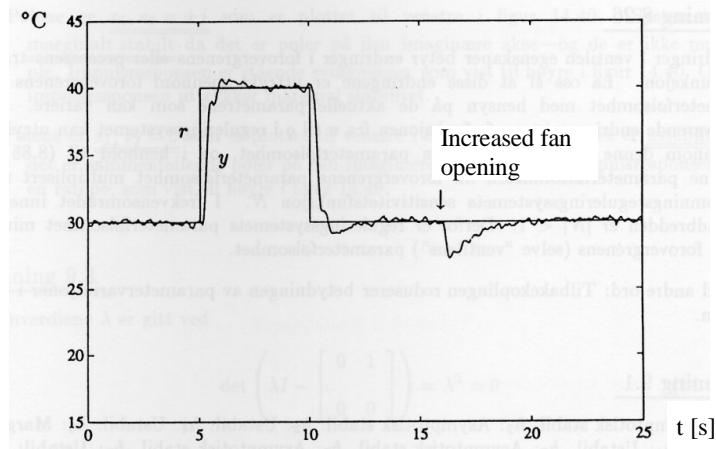


Figure 12.14:

Solution to Exercise 7.8

$$\underline{T_f} = 0.1T_d = 0.1 \cdot 2 = \underline{0.2 \text{ s}} \quad (12.102)$$

Solution to Exercise 7.9

The control signal is

$$u_d = K_p T_d \frac{d(\text{Setpoint step} - \text{constant})}{dt} \quad (12.103)$$

The time-derivative of a setpoint step is an *impulse*, which is a signal of infinite amplitude and with infinite duration, see Figure 12.15. So, the

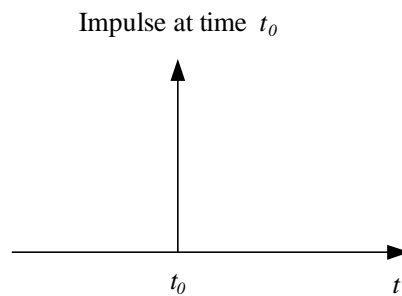


Figure 12.15:

stepwise change of the setpoint causes an impulse-like change of the control signal. That is the *derivative kick*.

Solution to Exercise 7.10

Under normal conditions the control error of PC₂ is

$$e_2 = SP_1 - SP_2 = 4 - 2 \text{ bar} = 2 \text{ bar} \quad (12.104)$$

This sustained non-zero control error would have caused the integral term of controller PC₂ to wind up – or, actually, to wind down to a very large negative value, making the controller virtually inactive if the pressure rises and comes close to SP₂, which makes the controller useless for limiting control.

Solution to Exercise 7.11

Reduced PB value means increased controller gain, and therefore increased loop gain. This implies that the stability is reduced if the PB value is decreased.

Solution to Exercise 7.12

1. The transport time from tank to sensor increases, thus increasing the time-delay in the control loop, causing reduced stability. If the time-delay gets too large, the control system becomes unstable.
2. With reduces flow, the time-delay is increased, causing reduced stability.
3. The stability is reduced since the loop gain is increased.
4. The stability is reduced since the loop gain is increased.

Solution to Exercise 7.13

The response is oscillatory because the control system is unstable. The instability is due to: (1) The reduced air flow which causes *increased transport delay (time-delay) in the process* from heating element to temperature sensor. (2) *Increased process gain* because the reduced air flow makes the temperature more sensitive to the supplied heat (adjusted by the control signal), and increased process gain implies increased control loop gain.

Solution to Exercise 8.1

We substitute x by its reference x_r and then solve for the control variable F_p which we denote the feedforward control variable F_{pf} . The result is

$$\underline{\underline{F_{pf} = m\ddot{x}_r - D|\dot{x}_r - u_c|(\dot{x}_r - u_c) + F_w}} \quad (12.105)$$

To implement the feedforward controller: x_r must be known, no problem. Parameters m and D must be known. Their values should be known by the ship designer. Water current u_c may be measured. However, in real DP applications (Dynamic Positioning) u_c is estimated using a state estimator algorithm named Kalman Filter. Wind force F_w must be known. In real DP applications it is calculated from measured wind direction and wind speed using a so-called “wind model”. (The sensor for measuring wind direction and speed is placed on the top of the ship.)

Solution to Exercise 8.2

We substitute the temperature T by the temperature setpoint T_{SP} (the time argument t is omitted for simplicity, but it should not be omitted if the model contain time-delay terms):

$$c\rho V\dot{T}_{SP} = K_h u + cw(T_{in} - T_{SP}) + U(T_e - T_{SP}) \quad (12.106)$$

We solve (12.106) for the control variable u to get the feedforward control variable u_f :

$$\underline{\underline{u_f}} = \frac{1}{K_h} [c\rho V\dot{T}_{SP} - cw(T_{in} - T_{SP}) - U(T_e - T_{SP})] \quad (12.107)$$

$$= \underbrace{\frac{1}{K_h} [c\rho V\dot{T}_{SP} + cwT_{SP} + UT_{SP}]}_{u_{fSP}} + \underbrace{\frac{1}{K_h} [-cwT_{in} - UT_e]}_{u_{fd}} \quad (12.108)$$

Implementation of feedforward control signal u_f requires measurement or knowledge of the following five quantities: c , ρ , V , h , w , K_h and T_{in} , in addition to the setpoint time-derivative, \dot{T}_{SP} .

Solution to Exercise 8.3

With the two data points given, we have

$$52 = a \cdot 40 + b \quad (12.109)$$

and

$$61 = a \cdot 60 + b \quad (12.110)$$

From these two equations we get (the mathematics to solve these two equations is not shown here)

$$\underline{\underline{a = 0.45}} \quad (12.111)$$

$$\underline{\underline{b = 34}} \quad (12.112)$$

Solution to Exercise 9.1

The control signal (current) is sent through a resistor of resistance

$$R = \frac{10 \text{ V}}{20 \text{ mA}} = 500 \Omega \quad (12.113)$$

The voltage drop across the resistor is used to control the device, see 12.16.

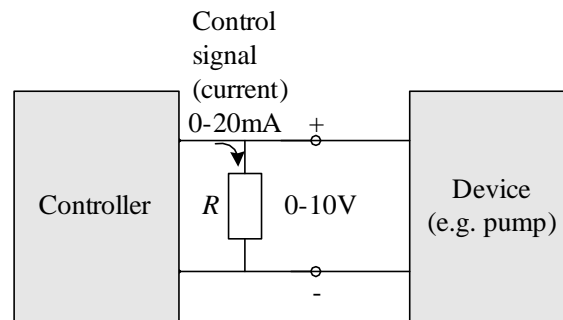


Figure 12.16:

Solution to Exercise 9.2

In line 6 from the top: “Control action, Direct, reversed”.

Solution to Exercise 9.3

$$\underline{\underline{DC}} = \frac{0.1}{5} \cdot 100 = \underline{\underline{2\%}} \quad (12.114)$$

$$\underline{\underline{T_{on}}} = 2\% \cdot 1 \text{ min} = \underline{\underline{0.02 \text{ min} = 1.2 \text{ s}}} \quad (12.115)$$

Solution to Exercise 10.1

See Figure 12.17. We read off

$$T_{ou} = 8.0 \text{ min} \quad (12.116)$$

which gives

$$\underline{\underline{T_i}} = 1.5T_{ou} = \underline{\underline{12.0 \text{ min}}} \quad (12.117)$$

And we can try keeping

$$\underline{\underline{K_p}} = 0.8 \cdot 4.0 = \underline{\underline{3.2 \text{ min}}} \quad (12.118)$$

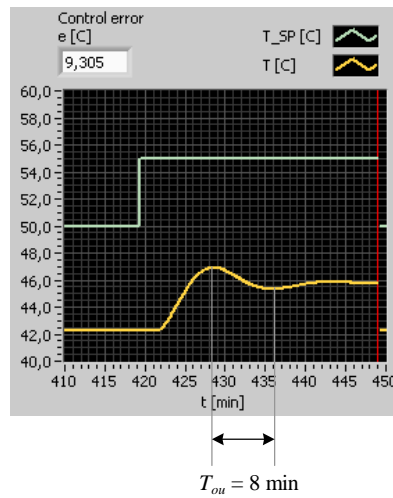


Figure 12.17:

If it turns out that the stability of the control system is too bad with this value of K_p , you can try reducing the gain and/or increasing the integral time.

Solution to Exercise 10.2

We use

$$T_C = \tau = 1 \text{ s} \quad (12.119)$$

According to Skogestad's tuning formulas for "time-constant with time-delay":

$$\underline{\underline{K_p}} = \frac{T}{K(T_c + \tau)} = \frac{5}{0.5 \cdot (1 + 1)} = \underline{\underline{5}} \quad (12.120)$$

$$\underline{\underline{T_i}} = \min[T, c(T_c + \tau)] = \min[5, 2(1 + 1)] = \min[5, 4] = \underline{\underline{4 \text{ s}}} \quad (12.121)$$

$$\underline{\underline{T_d}} = 0 \quad (12.122)$$

(Above, $\min[x, y]$ means the minimum of the two numbers x and y .)

Solution to Exercise 10.3

1. The Laplace transform of (10.5) is

$$\rho A [s y(s) - y_0] = K_s e^{-\tau s} u(s) - w_{out}(s) \quad (12.123)$$

Solving for output variable h gives

$$y(s) = \frac{1}{s}h_0 + \underbrace{\frac{K_s}{\rho A s} e^{-\tau s} u(s)}_{H_1(s)} + \left(-\frac{1}{\rho A s}\right) w_{out}(s) \quad (12.124)$$

Thus, the transfer function from u to y is

$$H_1(s) = \frac{y(s)}{u(s)} = \frac{K_s}{\rho A s} e^{-\tau s} \quad (12.125)$$

The Laplace transform of (10.6) is

$$H_2(s) = \frac{y_m(s)}{y(s)} = K_m \quad (12.126)$$

Combining these two transfer functions gives

$$\underline{\underline{H(s) = \frac{y_m(s)}{u(s)} = \frac{y_m(s)}{y(s)} \cdot \frac{y(s)}{u(s)} = \frac{K_s K_m}{\rho A s} e^{-\tau s} = \frac{K}{s} e^{-\tau s}}} \quad (12.127)$$

where

$$K = \frac{K_s K_m}{\rho A} \quad (12.128)$$

$H(s)$ is “integrator with time-delay”.

2. We set

$$T_C = \tau \quad (12.129)$$

According to Skogestad’s tuning formulas for “integrator with time-delay”:

$$\underline{\underline{K_p}} = \frac{1}{K (T_c + \tau)} = \frac{1}{\frac{K_s K_m}{\rho A} (2\tau)} = \frac{0.5 \rho A}{\underline{\underline{K_s K_m \tau}}} \quad (12.130)$$

$$\underline{\underline{T_i}} = c(T_c + \tau) = 2(2\tau) = \underline{\underline{4\tau}} \quad (12.131)$$

$$\underline{\underline{T_d}} = 0 \quad (12.132)$$

Solution to Exercise 10.4

From Figure 10.3 we may say that the process looks like “time-constant with time-delay”. The gain becomes

$$K = \frac{y_{\text{steady_state}}}{U} = \frac{10}{2} = 5 \quad (12.133)$$

The time-delay is (approximately)

$$\tau \approx 1 \text{ sec} \quad (12.134)$$

The time-constant is (approximately) the time from the end of the time-delay to the 63 % rise time:

$$T \approx 2 \text{ sec} \quad (12.135)$$

According to Skogestad's PID setting formulas for "time-constant with time-delay" (with $T_c = \tau$):

$$\underline{\underline{K_p}} = \frac{T}{K(T_c + \tau)} = \frac{2}{5 \cdot (1 + 1)} = \underline{\underline{0.2}} \quad (12.136)$$

$$\underline{\underline{T_i}} = \min[T, c(T_c + \tau)] = \min[2, 2(1 + 1)] = \min[2, 4] = \underline{\underline{2 \text{ s}}} \quad (12.137)$$

$$\underline{\underline{T_d}} = 0 \quad (12.138)$$

Solution to Exercise 10.5

1. In general closed loop-excitation is preferable because then the process is being controlled, which implies that the process variable will vary relatively about the setpoint. In other words, the process will stay around the operating point, which is good.
2. Most tanks are integrators dynamically, which implies that a constant control signal will cause a ramped response in the process output, bringing the response far off the operating point. For such processes closed loop excitation is preferable.

Solution to Exercise 10.6

For best control performance: B. Simplest: A.

Solution to Exercise 10.7

1. The stability would be decreased.¹
2. Let us indicate initial values (before the change) with index 0, and new values (after the changes) with index 1. According to Skogestad's table on page 142 in the text-book,

$$K_{p0} = \frac{T_0}{K_0(T_C + \tau_0)} \quad (12.139)$$

¹For processes that are unstable, the controller gain must actually be "large" for the control system to be stable, but such processes are relatively rare (one example is exothermal reactors).

$$T_{i_0} = \min[T_0, c(T_C + \tau_0)] \quad (12.140)$$

and, of course,

$$K_{p_1} = \frac{T_1}{K_1(T_C + \tau_1)} \quad (12.141)$$

$$T_{i_1} = \min[T_1, c(T_C + \tau_1)] \quad (12.142)$$

If the process gain K is the only parameter that has changed, we get from (12.139) and (12.141)

$$\frac{K_{p_0}}{K_{p_1}} = \frac{\frac{T_0}{K_0(T_C + \tau_0)}}{\frac{T_1}{K_1(T_C + \tau_1)}} = \frac{\frac{T_0}{K_0(T_C + \tau_0)}}{\frac{T_0}{K_1(T_C + \tau_0)}} = \frac{K_1}{K_0} \quad (12.143)$$

from which we get the following formula for the new controller gain:

$$\underline{\underline{K_{p_1} = K_{p_0} \frac{K_0}{K_1}}} \quad (12.144)$$

For the integral time there will be no change because the process gain is not in the formula of T_i :

$$\underline{\underline{T_{i_1} = T_{i_0}}} \quad (12.145)$$

Solution to Exercise 10.8

From the table shown in Figure 10.4 we see that Gain = K_p has (should have) less value and Integral = T_i has (should have) larger value the lower the temperature. This indicates that minimum temperature is “worst case”. Therefore, a PID controller with fixed settings should be tuned at minimum temperature.

Solution to Exercise 10.9

$$\underline{\underline{K_p}} = \frac{0.5 - 0.4}{30\% - 20\%} (GS - 20\%) + 0.4 = \underline{\underline{0.01\%^{-1} \cdot GS + 0.2}} \quad (12.146)$$

Solution to Exercise 10.10

The table-lookup is used straightforward: The input to the table-lookup function is z_{PID} which is the z -value that the PID controller demands, and the output from the table-lookup function is u which is used as control signal to the nonlinear process part (e.g. valve).

Solution to Exercise 11.1

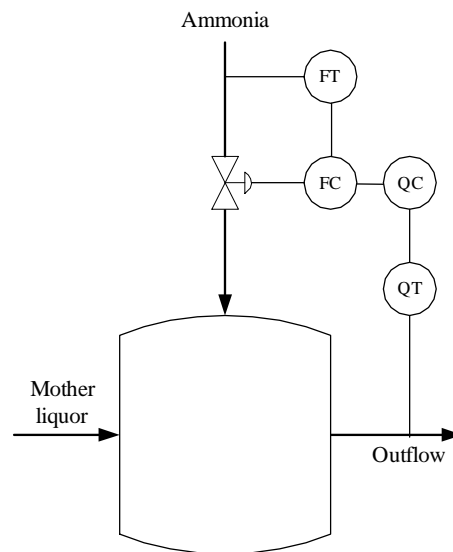


Figure 12.18:

See Figure 12.18.

The purpose of the ammonia flow control loop can be to obtain an ammonia flow that tracks the flow value (flow setpoint) that the pH controller demands. The flow control will compensate for flow variations caused by e.g. pressure variations in the ammonia gas supply.

Solution to Exercise 11.2

Figure 12.19 shows the instrumentation diagram.

Solution to Exercise 11.3

Figure 12.20 shows the ship with control system.

The purpose of the position control loop is to make the ship track the position reference.

The purpose of the propeller speed control loop is to make the propeller speed track the speed command or reference generated by the positional controller. The speed control loop will compensate for disturbances acting on the propeller so that the propeller speed is more smooth. In addition, this speed control loop may make the tuning of the position controller easier because the position controller will control the speed more directly.

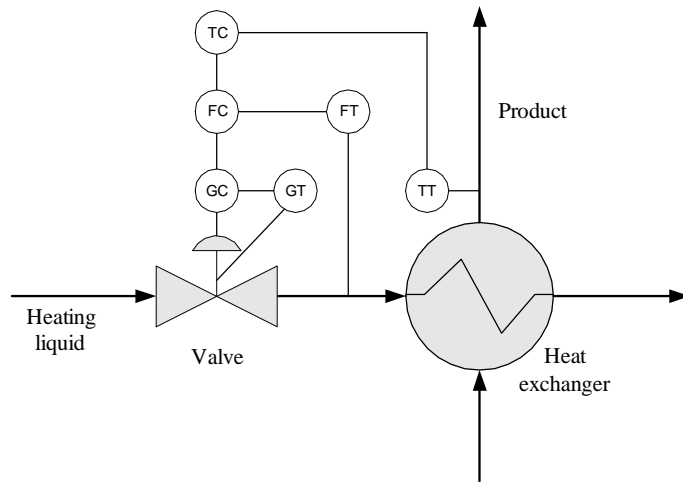


Figure 12.19:

Solution to Exercise 11.4

See Figure 12.21.

Solution to Exercise 11.5

Figure 12.22 shows the pH control system.

Solution to Exercise 11.6

1. Substituting F_{out} in the process model by F_{out} from the controller gives the following model of the control system:

$$\rho A \dot{h} = F_{in} - F_{out} = F_{in} - K_p(h_{SP} - h) \quad (12.147)$$

Assuming static conditions the model is

$$0 = F_{in_s} - K_p(h_{SP_s} - h_s) \quad (12.148)$$

which gives

$$\underline{\underline{h_s = h_{SP_s} + \frac{1}{-K_p} F_{in_s}}} \quad (12.149)$$

2. From (12.149) we see that the difference between the level and level setpoint is

$$\Delta h_s = e_s = h_{SP_s} - h_s = \frac{1}{K_p} F_{in_s} \quad (12.150)$$

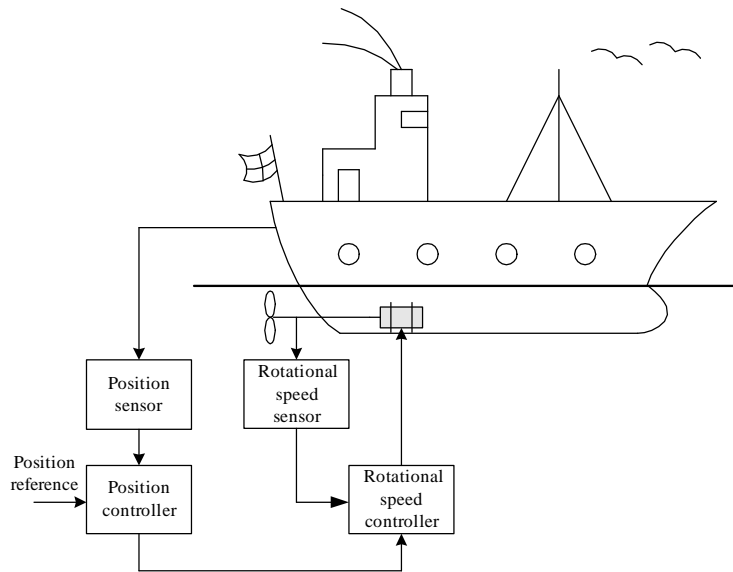


Figure 12.20:

Solving for K_p :

$$K_p = \frac{1}{\Delta h_s} F_{in_s} = \frac{1}{e_s} F_{in_s} \quad (12.151)$$

Δh_s is reduced if K_p is increased.

3. The drawback about using a P controller is that the static control error $e_s = \Delta h_s$ becomes different from zero.

Solution to Exercise 11.7

See Figure 12.23.

Solution to Exercise 11.8

See Figure 12.24.

Solution to Exercise 12.1

Figure 12.25 shows the Sequential Function Chart (SFC) solving the control task given above.

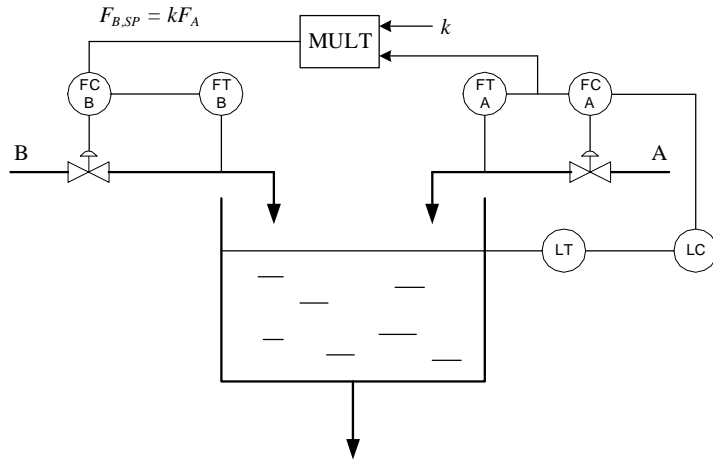


Figure 12.21:

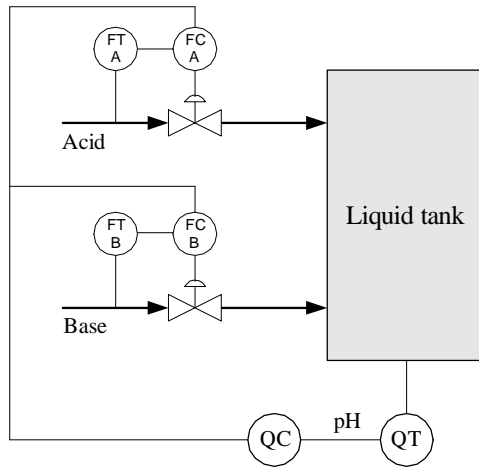


Figure 12.22:

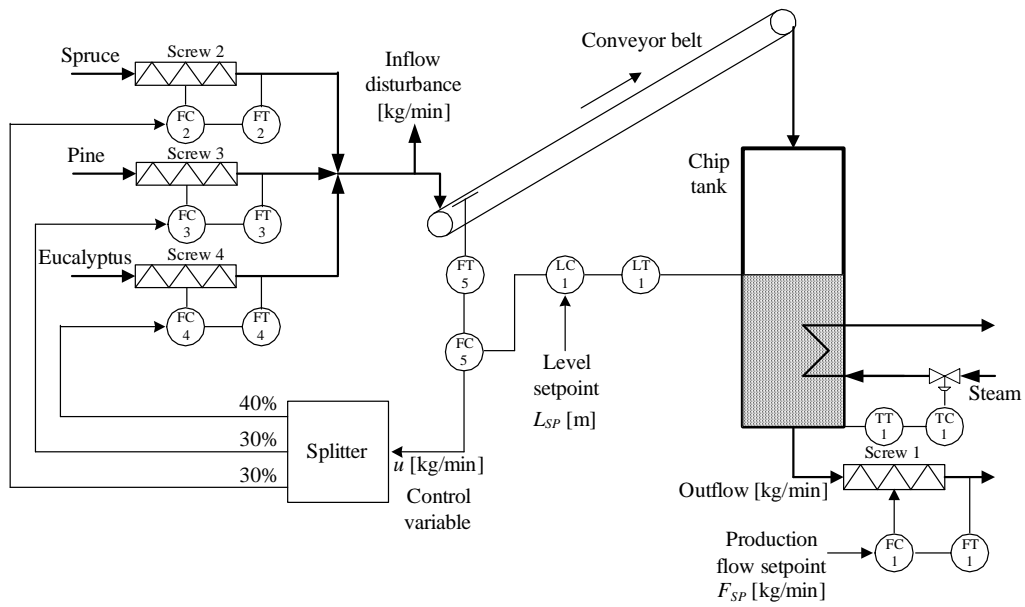


Figure 12.23:

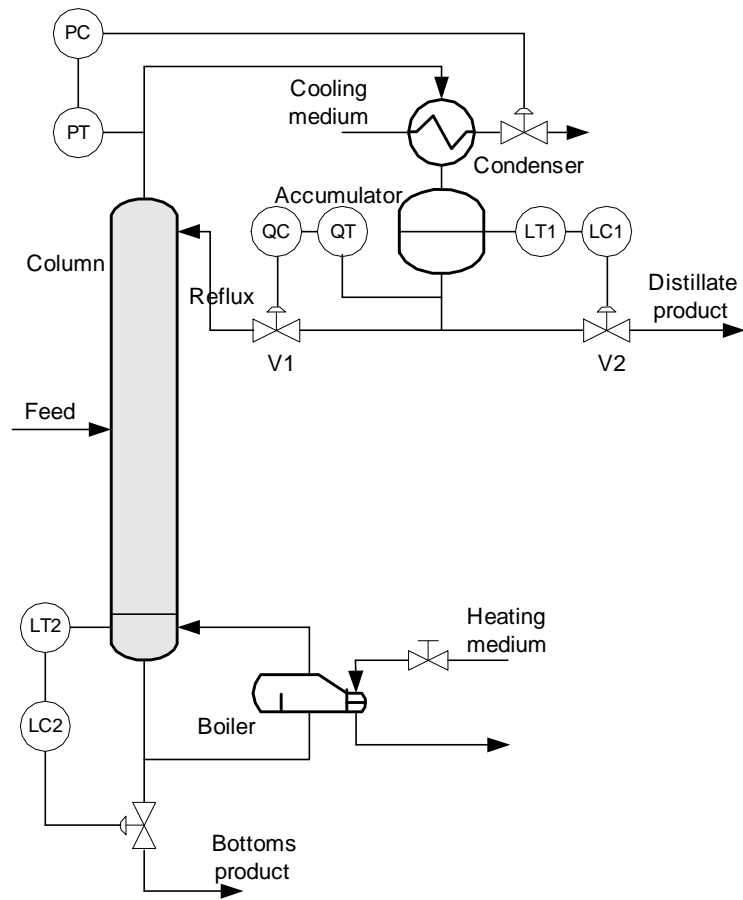


Figure 12.24:

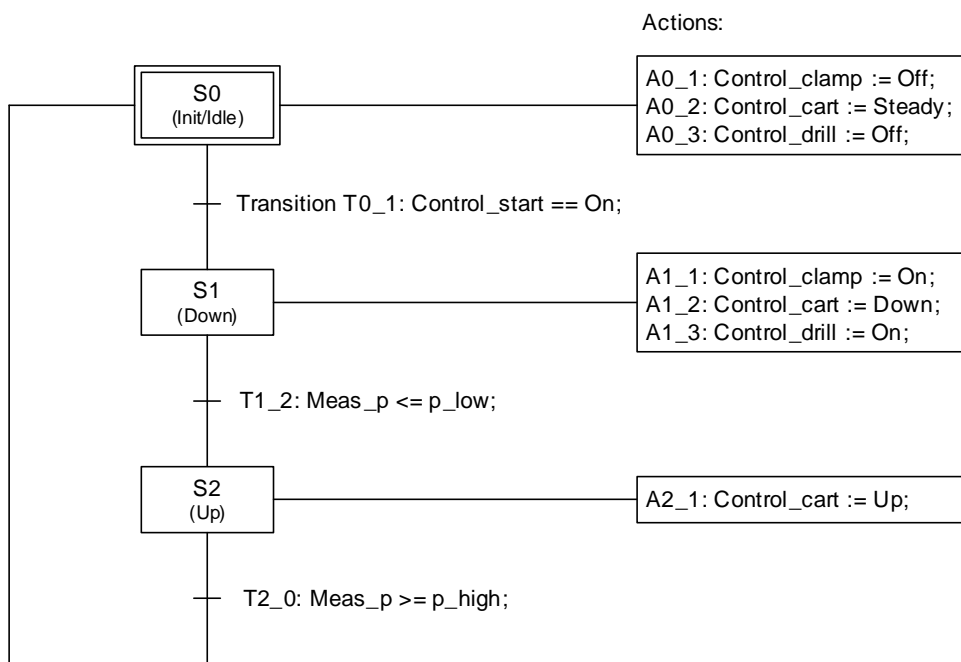


Figure 12.25: