**Problem 1a.** The system can reach steady oscillation when the Kp=26, Ti=3000000, and Td=0, as is shown in Figure 1.

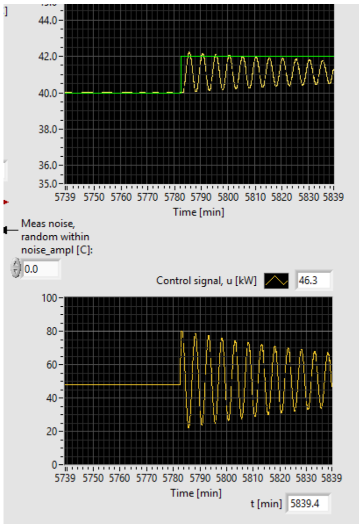



*Figure 1 The system almost reach steady oscillation when the PID setting is Kp=26, Ti=3000000, and Td=0.*

Therefore, the uilimate gain $K_{Pu}$=26, and the period Pu=300 s. According to Ziegler-Nichols' method, the PI controller can be Kp=0.45*26=11.7, Ti=300/1.2=250. When there is a step change of the set-point, the response of the system is shown in Figure 2. It took some time reach the new set-point steadily, but the result is acceptable because the amplitude decay ratio A1/A2 is almost equal to ¼.

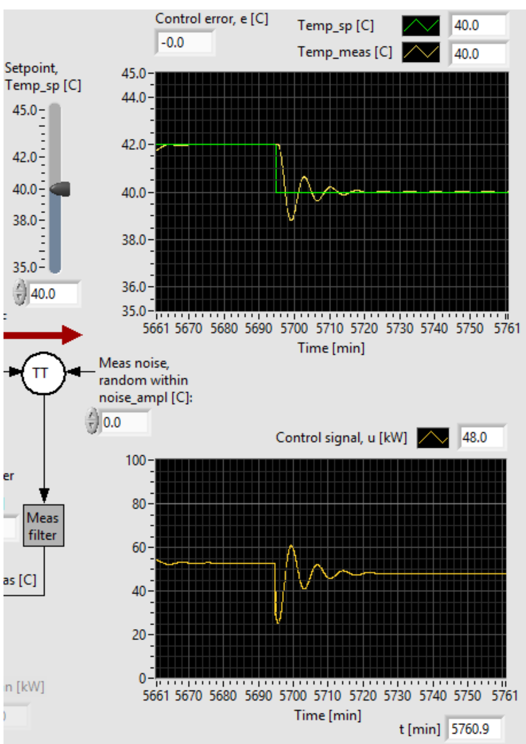



*Figure 2 The system response to step change of set-point when applied a PI controller tuned according to Ziegler-Nichols' method.*

When there is a step change of ambient temperature, the behavior of the system applying the same PI controller can be found in Figure 3.

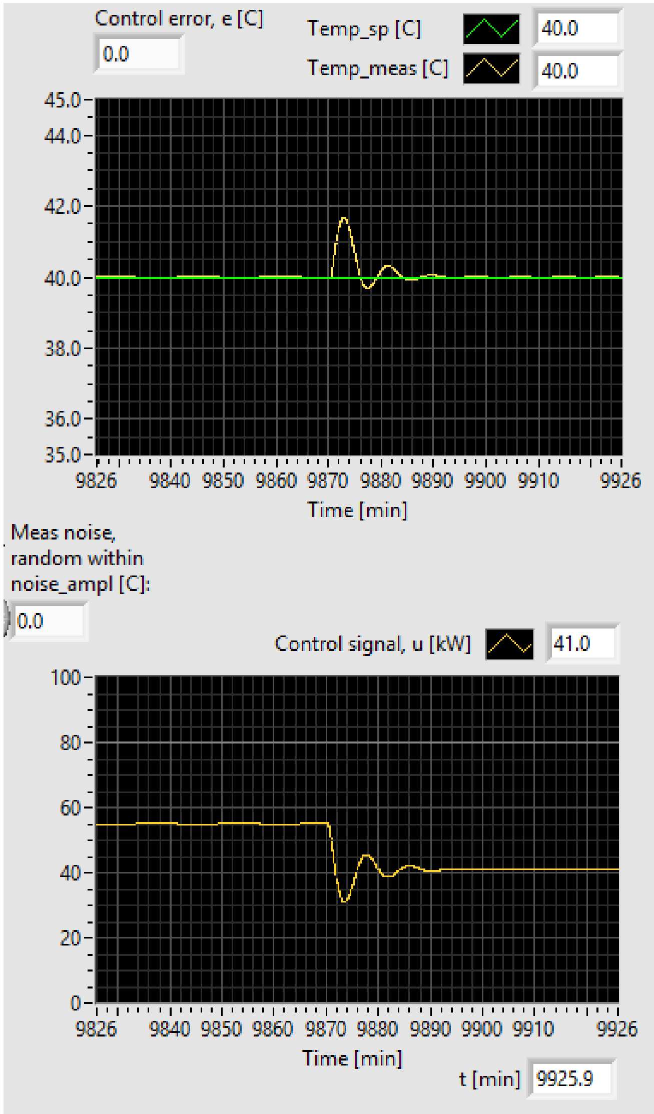



*Figure 3 Response to step change of disturbance when applying the same PI controller tuned according to Ziegler-Nichols' method.*

## Problem 2b

When using the Relaxed Ziegler-Nichols' method, Kp=0.32*$K_{Pu}$=0.32*26=8.32 and Ti=Pu=300.

When applying this PI controller, the system response to a step change of set-point and disturbance are shown in Figure 4. The temperature will reach the set-point and become stabilizing in a short period. The PI controller tuned based on Relaxed ZN PI-tuning can stabilize the system faster that of conventional Ziegler-Nichols' method.

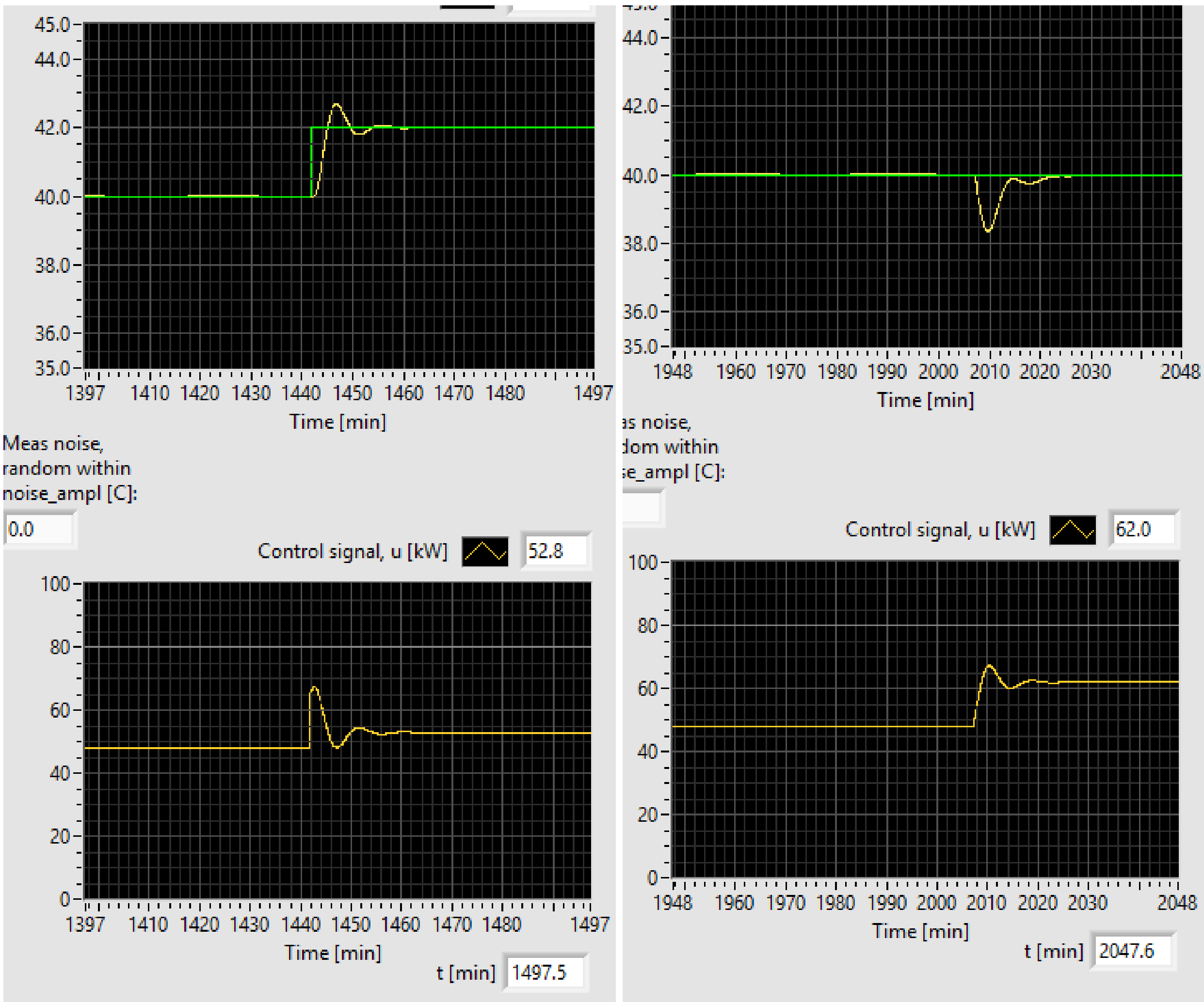



*Figure 4 System response to step change when applying PI controller with Kp=8.32 and Ti=300.*

## Problem 1c. The relay method

When the system is in steady state, use on/off controller with amplitude of A= 20, and the control signal oscillation amplitude will be Y= 1.2. Then the ultimate gain Kpu=4*A/(pi*Y)=21.2, and the period Pu= 330

Now applying the Zn method

Kp=0.45*Kpu=0.45*21.2=9.54;

and Ti=Pu/1.2=330/1.2=275

The result was shown in Figure 5, the performance was slightly better than using conventional ZN method.

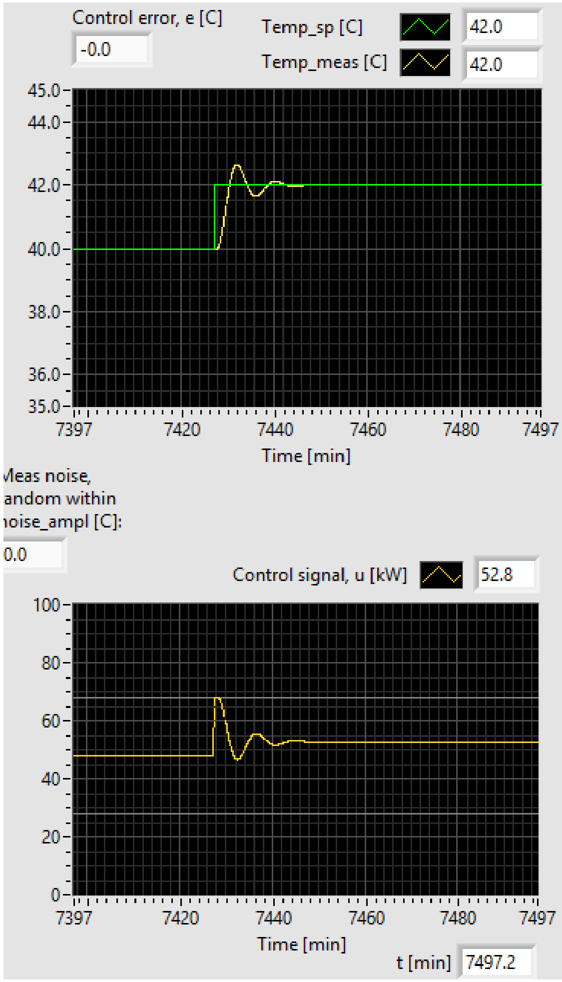



*Figure 5 System performance when applying PI controller tuned based on relay method.*

## Problem 1d Use the Skogestad method

Step in the control signal in manual control mode, process time constant Tc and time delay tau can be found out.

In this case, change the input u from 48 to 58, the output temperature will change from 40 C to 44.2C. Assuming Tc= tau=30 sec. The 63% of total changing of temperature 2.65 c.

Therefore, tau=30, Tc=30, Ki=2.65/(10*30)=0.00883

Kp=1/[Ki*(Tc+tau)]=1.9, Ti=2(tau+Tc)=120

By applying the PI controller tuned according to Skogestad method, the step response test results are shown in Figure 6.

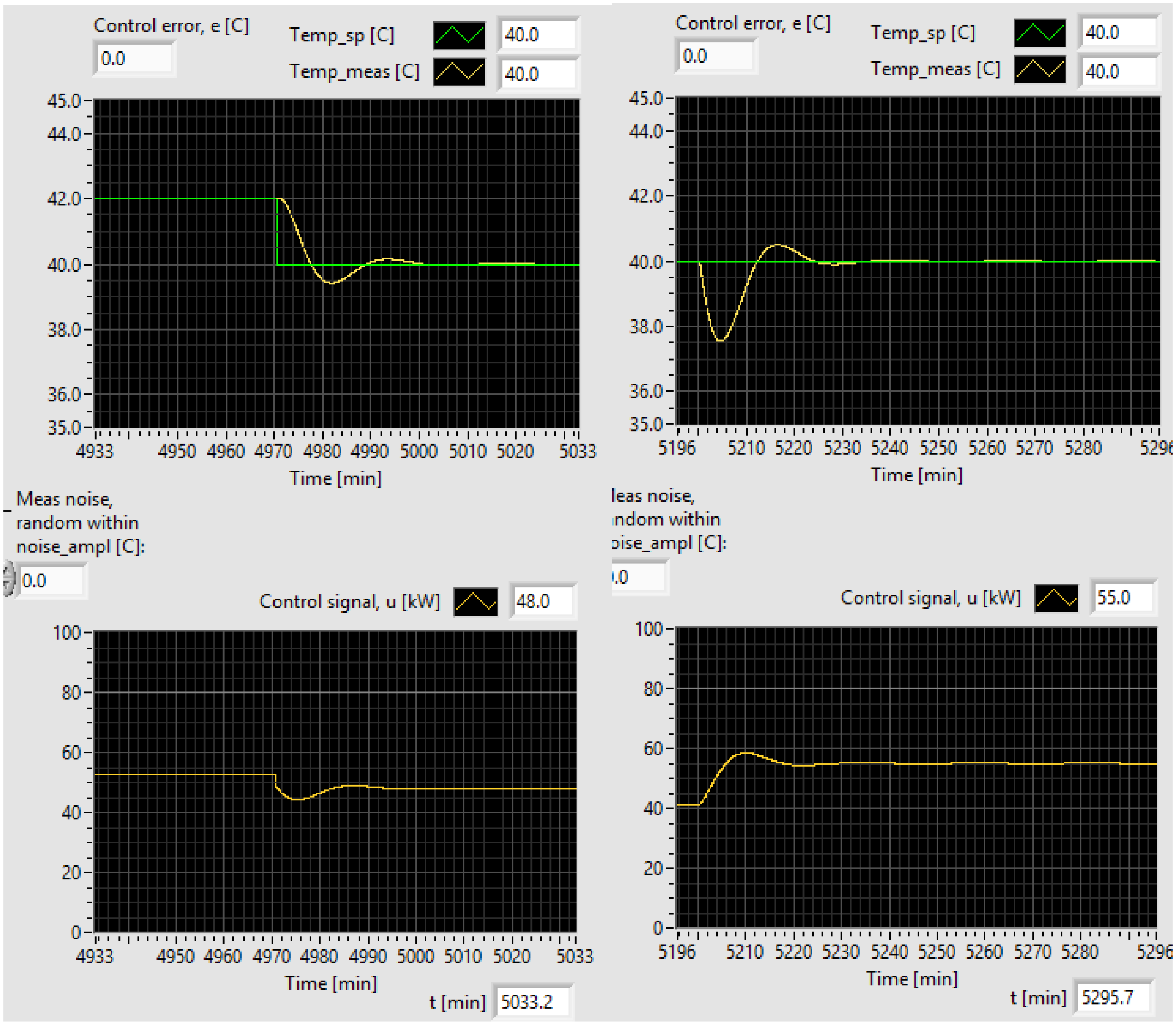
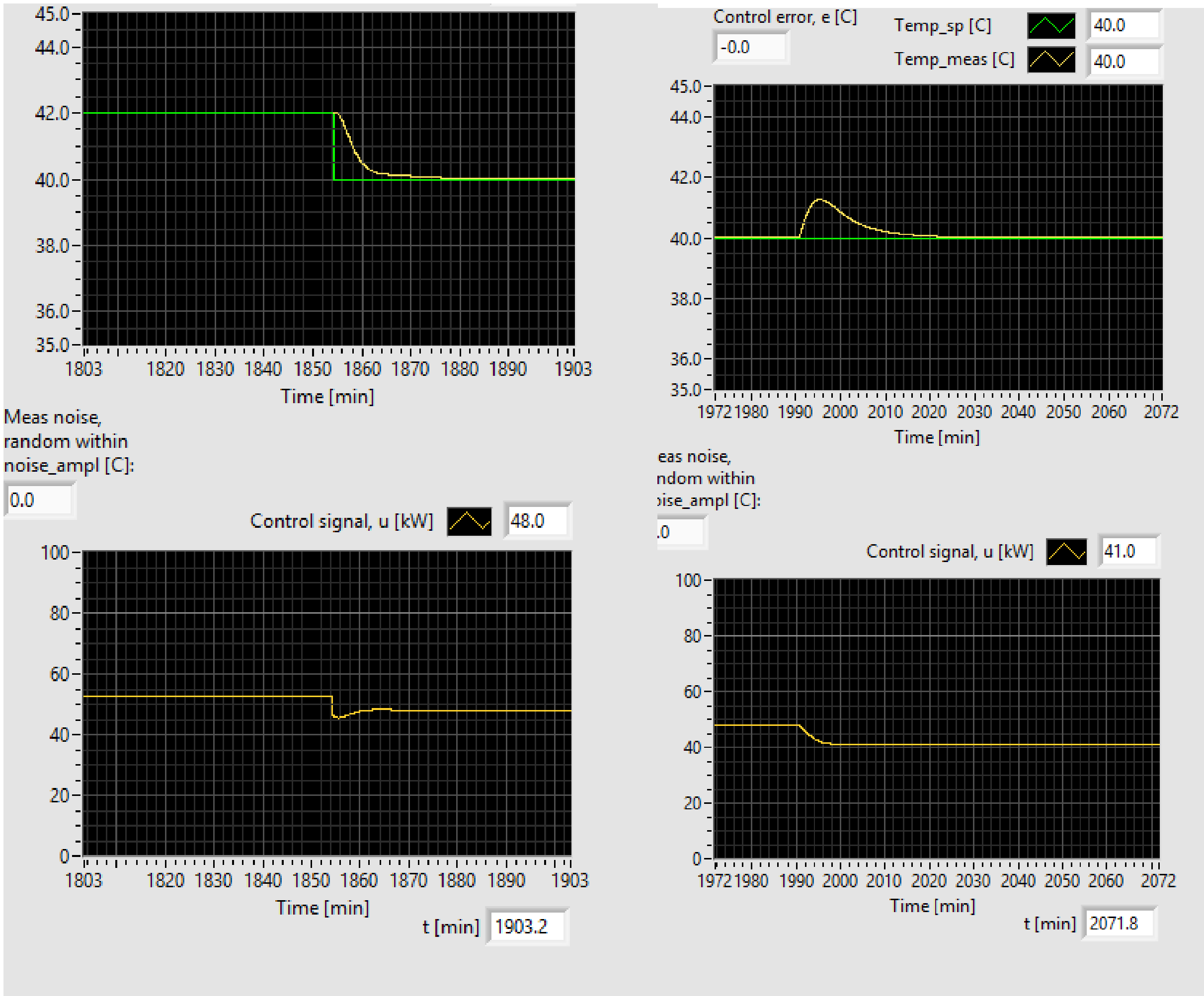



*Figure 6 Results of step respond when applying PI controller tuned according to Skogestad method.*




## Problem 1e Gain margin (GM) and phase margin (PM)

The GM and PM were calculated according to Haugen (2012). An adjustable gain $\Delta K$ and an adjustable time-delay $\Delta\tau$ were insert into the control loop. By changing $\Delta K$ or $\Delta\tau$ to bring the response to sustained oscillation, the GM and PM can be calculated as:

GM=$\Delta K$;

$$PM = 360 \cdot \frac{\Delta\tau}{P_{osc}}$$

The GM and PM for Ziegler-Nichols' tuning (Problem 1a) and Skogestad tuning (Problem 1d) are listed as below:

| | Ziegler-Nichols' tuning | Skogestad tuning |
|---|---|---|
| Kp | 11.7 | 1.9 |
| Ti | 250 s | 120 s |
| $\Delta K$ | 1.71 | 6.32 |
| $\Delta\tau$ | 23 | 110 |
| $P_{osc}$ | 240 s | 1140 s |

| | | |
|---|---|---|
| GM | 1.71 | 6.32 |
| PM | 34.5 | 34.7 |

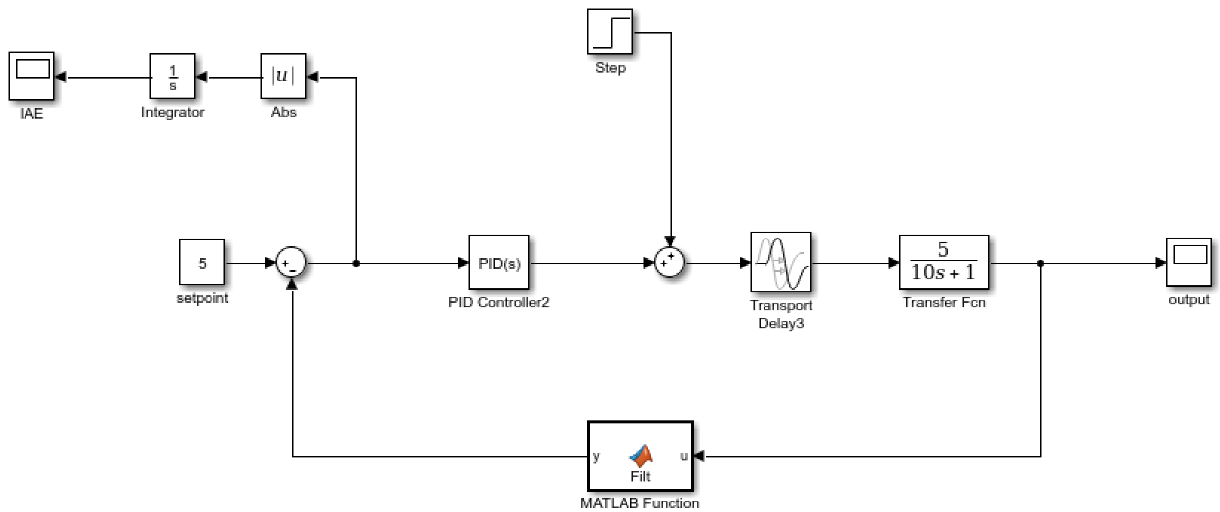
The ranges for acceptable values of stability margins (Seborg et al 2014):

$$1.7 = 4.6\,dB \leq GM \leq 12.0\,dB \leq 4.0$$

$$30^\circ \leq PM \leq 45^\circ$$

For the Ziegler-Nichols's tuning, the GM is on the lower limit, but the PM is acceptable. However, for Skogestad tuning, the GM is larger than the upper limit, but the PM is acceptable.

## Problem 1a



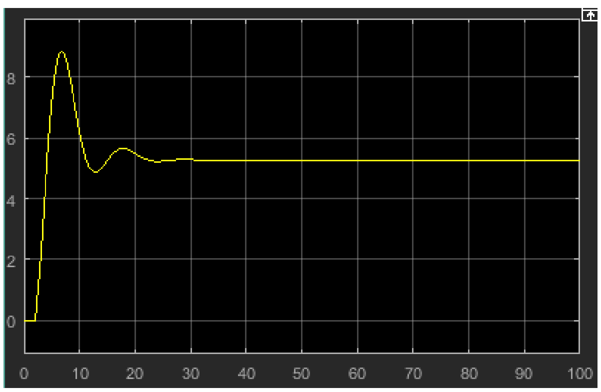

See the Simulink file. The system was assumed to have

## Problem 2b Using Ziegler-Nichols's method to tune a PI controller.

When the system reach steady oscillation, Kpu=1.87, Pu= 8 s.

Kp=0.45 Kpu=0.84, Ti=Pu/1.2=6.67, I=1/Ti=0.15

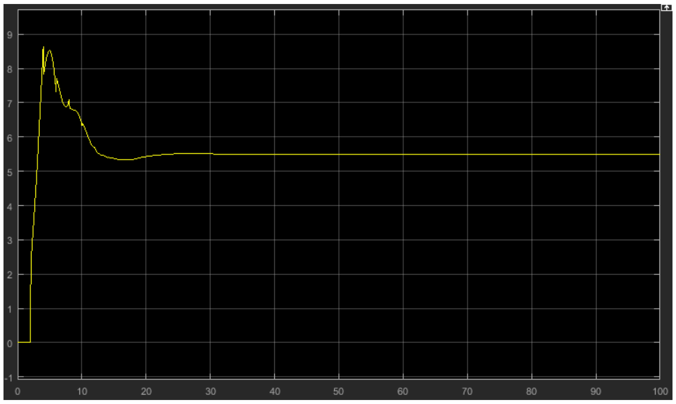
The system output is shown as below



The IAE value was 32.68 when the response has become constant.

## Problem 2c Using Ziegler-Nichols's method to tune a PID controller.

Kp=0.6 Kpu=1.12, I=1/Ti=2/Pu=0.25, Td=Pu/8=1



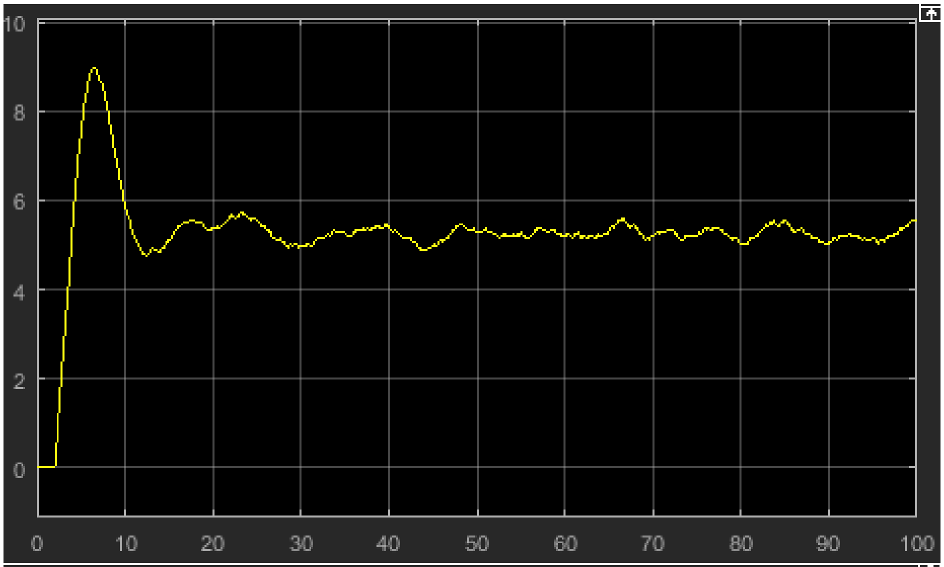

The IAE value is 24.77 when applying the PID controller, smaller than that of PI controller.

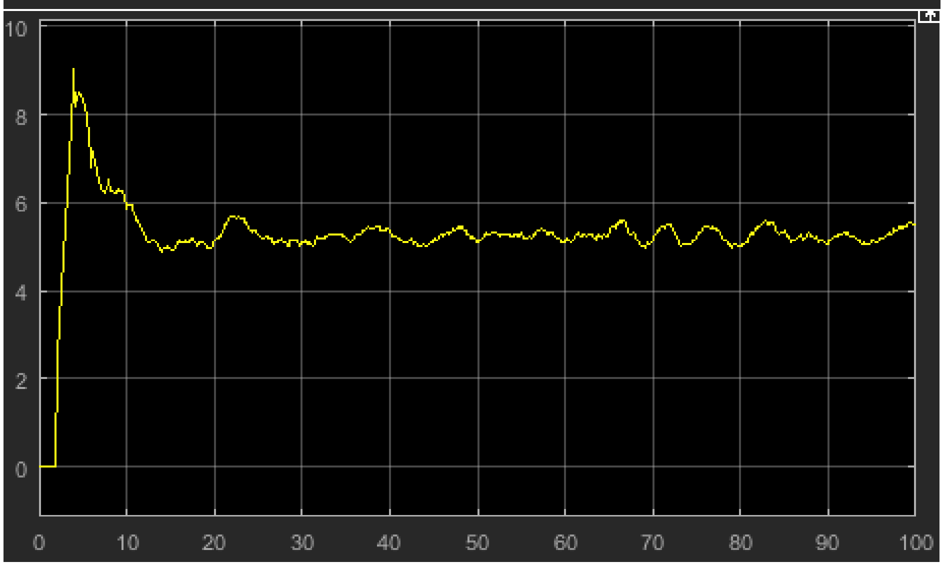
## Problem 2d Why PI often is preferred to PID in a practical system, even though the IAE index of PID is smaller than PI controller?

To answer this question, a simulation using uniform random signal as disturbance was applied to show the difference of PID and PI controller.

As is shown in the figure below, the output response of PID controller has a relatively smaller error margin, which lead to smaller IAE index. However, the PID controller also fluctuated significantly in short periods. In practice, the actuator will have to change very fast and very often due to the D term, and the life-length of the actuator will be reduced.

Even though PI controller respond slower than PID controller, the disturbance in practice is changing not very fast. A PI controller will be sufficient for most industry process.


10
8
6
4
2
0
0
10
20
30
40
50
60
70
80
90
100


10
8
6
4
2
0
0
10
20
30
40
50
60
70
80
90
100