Exercise 0.1 ***Sampled signal sequence***

Assume that a the ramp signal $r(t) = Rt$ is sampled with sampling frequency $f_s = 10\text{Hz}$ from time $t = 0$. Write the first five values of the resulting discrete-time signal $r_d(t_k)$.

Exercise 0.2 ***Writing a difference equation with only negative or zero time shifts***

Given the following difference equation:

$$y(k+3) + ay(k+1) = b_1 u(k+2) + b_0 u(k) \tag{1}$$

Write the corresponding difference equation having only zero or negative time shifts.

Exercise 0.3 ***Block diagram of a filtering algorithm***

The following difference equation is the filtering algorithm of a simple FIR[1] filter:

$$y(k) = \frac{1}{3}\left[u(k) + u(k-1) + u(k-2)\right] \tag{2}$$

Draw a block diagram of (2).

Exercise 0.4 ***Transfer function of a PI controller***

Here is the time-domain continuous-time PI controller function:

$$u(t) = \underbrace{K_p e(t)}_{u_p} + \underbrace{\frac{K_p}{T_i}\int_0^t e\,dt}_{u_i} \tag{3}$$

Using the Euler backward method for discretizing (3), we get the following discrete-time PI controller:

$$u(k) = u_p(k) + u_i(k) \tag{4}$$

where

$$u_p(k) = K_p e(k) \tag{5}$$

$$u_i(k) = u_i(k-1) + \frac{K_p h}{T_i} e(k) \tag{6}$$

Derive the PI controller transfer function (from e to u) from (4) – (6). Write the transfer function with positive exponents of z.

Exercise 0.5 ***Poles and zeros***

[1] FIR = Finite Impulse Response

Given the following transfer function:

$$H(z) = \frac{bz^{-2} + z^{-1}}{1 - az^{-1}} \tag{7}$$

Calculate the poles and the zeros of the transfer function.

Exercise 0.6 ***Stability analysis of numerical algorithm***

Discretizing the continuous-time transfer function

$$H_{\text{con}}(s) = \frac{y(s)}{u(s)} = \frac{K}{Ts + 1} \tag{8}$$

using Euler's forward method with time-step h yields the following discrete-time transfer function:

$$H_{\text{dis}}(z) = \frac{y(z)}{u(z)} = \frac{\frac{Kh}{T}}{z - \left(1 - \frac{h}{T}\right)} \tag{9}$$

For which (positive) values of h is the discrete-time system asymptotically stable? (You can assume that T is positive.)

Exercise 0.7 ***Setting up the sensitivity transfer function of a control system***

Figure 1 shows a block diagram of a feedback control system.

- What is the loop transfer function $L(z)$ of the control system?
- What is the sensivity transfer function $S(z)$?
- What is the tracking transfer function $T(s)$?

Exercise 0.8 ***Stability analysis in a Bode diagram***

Figure 2 shows the Bode curves of the loop transfer function of a given control system.

What are the stability margins GM and PM, and the crossover frequencies ω_c and ω_{180}?

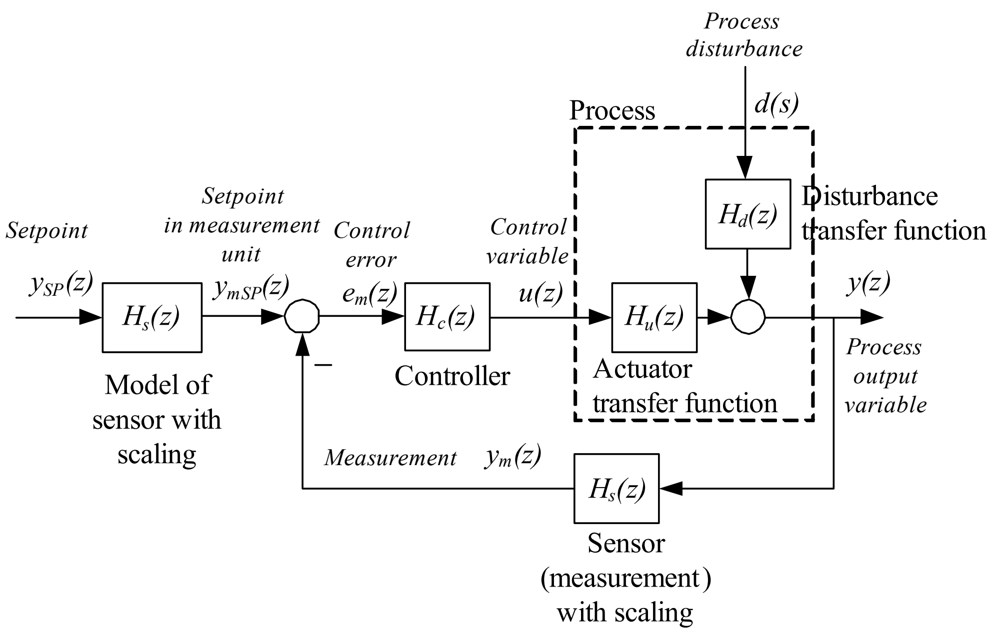

Figure 1: Exercise 0.7: Feedback control system

Solution 0.1

The sampling time or interval is

$$h = \frac{1}{f_s} = \frac{1}{10} = 0.1\text{s} \tag{10}$$

Since $t_k = kh$,

$$r_d(t_k) = Rt_k = Rkh \tag{11}$$

The first five values corresponds to $k = 0, .., 4$, i.e.

$$\{0,\ Rh,\ R2h,\ R3h,\ R4h\} \tag{12}$$

or

$$\{0,\ 0.1R,\ 0.2R,\ 0.3R,\ 0.4R\} \tag{13}$$

Solution 0.2

Each of the time indexes is reduced by 3, to give

$$\underline{\underline{y(k) + ay(k-2) = b_1 u(k-1) + b_0 u(k-3)}} \tag{14}$$

Solution 0.3

Figure 3 shows the block diagram.

Solution 3

Taking the z-transform of (4) – (6) gives

$$u(z) = u_p(z) + u_i(z) \tag{15}$$

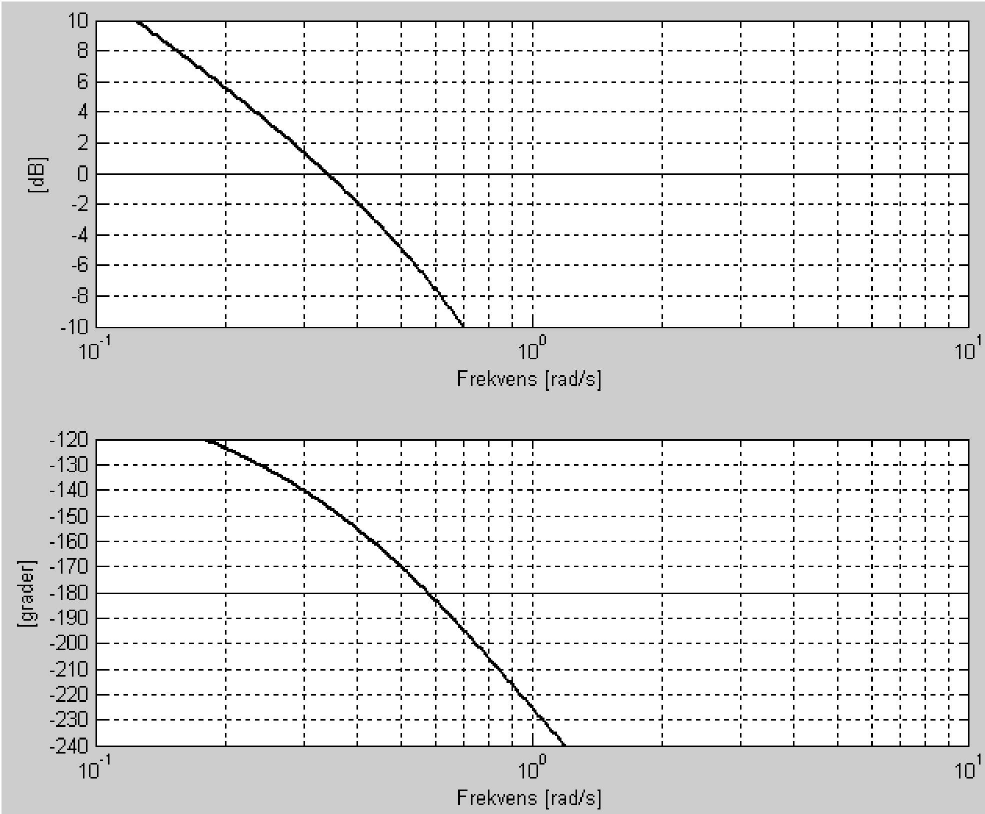

Figure 2: Exercise 0.8: Bode plots of control system

where

$$u_p(z) = K_p e(z) \tag{16}$$

$$u_i(z) = z^{-1} u_i(z) + \frac{K_p h}{T_i} e(z) \tag{17}$$

From (17) we solve for $u_i(z)$ to get

$$u_i(z) = \frac{K_p h / T_i}{1 - z^{-1}} e(z) \tag{18}$$

Inserting (16) and (18) into (15) yields

$$\begin{aligned} u(z) &= u_p(z) + u_i(z) && (19) \\ &= K_p e(z) + \frac{K_p h / T_i}{1 - z^{-1}} e(z) && (20) \\ &= \frac{(K_p + K_p h / T_i) - K_p z^{-1}}{1 - z^{-1}} e(z) && (21) \\ &= \frac{(K_p + K_p h / T_i)\, z - K_p}{z - 1} e(z) && (22) \end{aligned}$$

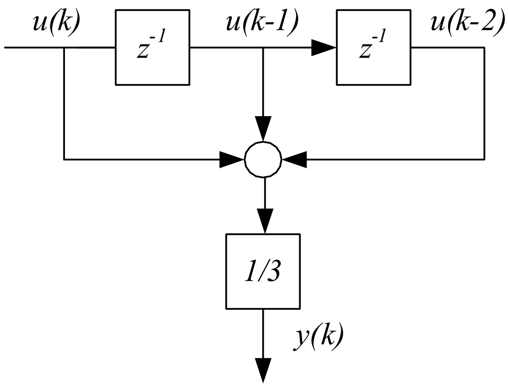

Figure 3: Solution 0.3: Block diagram

Thus, the transfer function is

$$\underline{\underline{H_c(z) = \frac{u(z)}{e(z)} = \frac{(K_p + K_p h/T_i)\, z - K_p}{z - 1}}} \tag{23}$$

Solution 0.5

It is convenient to start by rewriting the transfer function as follows:

$$H(z) = \frac{z^{-2}b + z^{-1}}{1 - az^{-1}} \cdot \frac{z^2}{z^2} = \frac{z + b}{z^2 - az} = \frac{z + b}{(z - a)\, z} \tag{24}$$

Thus, the zero ζ is

$$\underline{\underline{\zeta = -b}} \tag{25}$$

and the poles p_i are

$$\underline{\underline{p_1 = a;\ p_2 = 0}} \tag{26}$$

Solution 0.6

The pole of (9) is

$$p = 1 - \frac{h}{T} \tag{27}$$

The system is asymptotically stable if

$$\left| p = 1 - \frac{h}{T} \right| < 1 \tag{28}$$

If

$$1 - \frac{h}{T} > 0 \tag{29}$$

(28) becomes

$$1 - \frac{h}{T} < 1 \tag{30}$$

which gives

$$\underline{\underline{h > 0}} \tag{31}$$

which is always satisfied.

If

$$1 - \frac{h}{T} < 0 \tag{32}$$

(28) becomes

$$-\left(1 - \frac{h}{T}\right) < 1 \tag{33}$$

which gives

$$\underline{\underline{h < \frac{T}{2}}} \tag{34}$$

Solution 0.7

1. The loop transfer function is the product of the transfer functions in the loop:

$$\underline{\underline{L(z) = H_c(z)H_u(z)H_s(z)}} \tag{35}$$

2. The sensitivity transfer function is

$$\underline{\underline{S(z)}} = \frac{1}{1 + L(z)} = \underline{\underline{\frac{1}{1 + H_c(z)H_u(z)H_s(z)}}} \tag{36}$$

3. The tacking transfer function is

$$\underline{\underline{T(z)}} = \frac{L(z)}{1 + L(z)} = \underline{\underline{\frac{H_c(z)H_u(z)H_s(z)}{1 + H_c(z)H_u(z)H_s(z)}}} \tag{37}$$

Solution 0.8

From the Bode diagram shown in Figure 2 we read off

$$\underline{\underline{GM = 7\text{dB} = 2.2}} \tag{38}$$

$$\underline{\underline{PM = 35^\circ}} \tag{39}$$

$$\underline{\underline{\omega_c = 0.34\text{rad/s}}} \tag{40}$$

$$\underline{\underline{\omega_{180} = 0.58\text{rad/s}}} \tag{41}$$